

ABSTRACT

SITUATION THEORY: A SURVEY

Situation semantics was developed as an alternative to possible-worlds semantics. While possible-worlds semantics defines the informational content of sentences in terms of complete descriptions of the way the world is or might be, situation semantics defines the informational content of sentences in terms of partial worlds called situations. Situation theory is a novel informational ontology developed to give situation semantics a rigorous mathematical foundation. In reviewing the literature, we observed that although there are a few texts introducing situation theory in varying levels of detail and systematicity, there has not yet been published, until now, a general survey of the situation-theory literature that might introduce and guide future scholars. We note that a similar, if perhaps less-pressing need exists for a survey of the situation-semantics literature, but regret that we must leave such a survey for others.

Jacob Ian Lee
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INTRODUCTION

The Need for a Theory of Semantic Information

Information and information flows are ubiquitous and polymorphic. They are evident in circumstances as disparate as the seating arrangements of a fono in a Samoan village, the mating signals of *Photinus pyralis*, the vast heterogeneous computer network known as the internet, and protein bio-synthesis. Talk of information permeates the discourse of numerous academic and professional disciplines. But what is information?

A variety of sophisticated approaches to the understanding and measurement of information have been developed. The most successful theories have been quantitative in nature; they attempt to answer the question of how much information is contained in a given message or data set. These theories most famously include mathematical communication theory, commonly called information or coding theory, proposed originally by Claude Shannon (1948), and the algorithmic information theory of Andrey Kolmogorov, Gregory Chaitin and others¹. However, quantitative theories of information cannot tell us *what* information is contained in a message or data set, or how it can inform us about the world². For that, we need a theory of semantic information and a theory of information flow.

¹ Roughly speaking, in mathematical communication theory, the amount of information of a received message is inversely proportional to its probability. In algorithmic information theory, the information content of a collection of data is something like the size, in bits, of the smallest program that would output that data if executed. Algorithmic information content increases with the randomness of the data. Algorithmic information may be described as a measure of descriptive complexity. For very useful overviews to these and other mathematical theories of information, we recommend (Bevaud 2009), (Harremoës and Topsøe 2008) and (Grünwald and Vitányi 2008) and (Calude 2009).

² This is not to say that measures of semantic information cannot be explored. In the semantic tradition there have been many attempts at finding an appropriate measure of semantic information. One influential early attempt at this is (Bar-Hillel and Carnap 1953). Their approach, and most like them,

Many philosophers of information believe that there can be no information without data representation (Floridi 2011). Unfortunately, there has been in some places a tendency to conflate the information carried by those representations with the representations themselves. But as anyone who has worked within an organization knows, simply making sure that everyone gets and reads the same email announcement is not enough to guarantee that everyone has the same information. The semantic content of a message is not some fixed feature of its data representation—though there is a difficult and interesting relationship between the two. There is a long tradition of formal semantics in which the meanings of expressions (in natural language) are taken to be propositions³.

Received Views of Semantic Information

A direct approach to the semantics of expressions is to understand their references. Gottlob Frege took the reference of sentences to be its truth value, and the reference of component expressions to be their contributions to the truth value of the sentences in which they are embedded. For example, in the sentence

Manuel is a programmer.

the reference of the proper noun MANUEL is the object it denotes, and the reference of the predicate IS A PROGRAMMER is a characteristic function from objects to truth values, and the sentence as a whole has as its reference *true* if Manuel is indeed a programmer and has as its reference *false* if Manuel is not a

generally presumes a possible-worlds semantics in which the amount of the information of a sentence is—roughly speaking—measured by the number of possible worlds its truth excludes. Given some definition of the logical probability of different states of affairs, the information of a sentence is measured inversely by the logical probability of the truth of the proposition it expresses.

³ In the following discussion of intensions, we rely closely upon (Speaks 2011).

programmer. One might think at first that we have a satisfactory theory of meaning here, but there are problems. The first is that in sentences like

Selena believes that Manuel is a programmer.

which involve ascriptions of what are called *propositional attitudes*, the contribution of the clause MANUEL IS A PROGRAMMER to the truth value of the sentence cannot be its reference (which is, as we've seen, its truth value according to Frege), because the truth of whether or not Selena believes that Manuel is a programmer is logically independent of whether in fact Manuel is a programmer. Indeed, in propositional attitude ascriptions, the substitutions of logically equivalent expressions will not generally preserve their reference. To take a classic example, Clark Kent and Superman refer, let us say, to the same person and so are logically equivalent, but it is obviously possible that Lois Lane can believe that Superman can fly without also believing that Clark Kent can fly. Another perhaps more fundamental problem with this view is that any two sentences that happen to have the same truth value necessarily have the same reference. The standard resolution of these dilemmas, and others like them, has been to say that the components of a sentence have a *content* in addition to a reference, and that that content in some way determines reference. The content of sentence is seen to be a proposition.

The details of how this works out are a little tricky. There are several considerations. First, there is the efficiency of language. We cannot simply assign contents to different expressions, because those contents may depend on contexts of utterance. For example, first-person pronouns like I and WE do not have fixed references. Their references will depend, in various ways, upon various facts about the context of their utterance, such as who the speaker is. This suggests that

expressions must be associated with rules that assign contents to expressions based on their contexts of utterance. These rules are generally called *characters* in the literature. But this only handles a part of the problem, for the proper determination of the truth value of sentences cannot simply depend on the character and the context of their utterance. Sentences must be evaluated in terms, it is argued, of the possible ways the world could be. For example, if John says in 2011:

I will be only dust and bones by the year 2100.

we know that “I” refers to John, but that the truth of the proposition expressed by the sentence John is not given by the state of John at the time of his utterance, but of his state in the year 2100.

These ideas found a natural expression in possible-world semantics, where the contents assigned to expressions are called *intensions*, which are functions from circumstances of evaluation to reference. The intension of a sentence is a function from possible worlds to truth values. Roughly, a possible-worlds framework is a 4-tuple consisting of a non-empty set of possible worlds, an accessibility relation between possible worlds, a distinguished world called the actual world, and a valuation function assigning sets of worlds to each atomic sentence of a formal language. Possible worlds are complete state descriptions of the way a world could be. A world w' is accessible from a world w if every true proposition at w' is possibly true at w .

One problem with these measures, and indeed with the possible-worlds framework generally, is that distinct but logically necessary sentences are true in every possible world and therefore have the same intensions. This means, for example, that $1 + 1 = 2$ and $\text{PI IS AN IRRATIONAL NUMBER}$ must “mean” the same thing. And because they have the same content when necessarily true propositions

such as the two above are embedded in the same propositional attitude ascription, then they must have the same truth value, e.g.:

Homer believes that $1 + 1 = 2$.

iff

Homer believes that Pi is an irrational number.

Indeed, various arguments show that belief in any proposition entails belief in every necessary truth (Speaks 2011). Clearly this is unwelcome.

Content Measure

A related view originally developed by Bar-Hillel and Carnap (1953) is more directly related to information, and leads to measures of semantic information. State descriptions resolve every issue in a world (relative to a formal language L) and can be thought of as a conjunction of each atomic statement of L or its negation (but not both). For each state description there is a *content element* defined as its negation, which is the weakest disjunction not consistent with the state description. Bar-Hillel and Carnap identify the content of a sentence of L in terms of the state descriptions that sentence *excludes*. Necessary truths exclude no state description since every state description is compatible with it, while contradictions exclude every state description, since no state description can be compatible with it.

If we assume some probability measure pr on the atomic sentences of L then two content measures can be associated with a sentence:

$$cont(A) =_{df} 1 - pr(A)$$

and

$$inf(A) =_{df} -\log_2(pr(A))$$

There are many things that can—and have been said—about these measures. Here we simply want to point out two oddities about them. The first is that under most probability measures, the amount of information of any necessary truth is zero, since the probability that they are true is unity; and second is that contradictions have maximum information. Necessary truths have no content because they exclude no state descriptions, whereas contradictions exclude too many state descriptions.

What is Situation Theory and Situation Semantics?

Despite the many successes of possible-worlds semantics, and its widespread popularity, possible-worlds semantics faces some difficult problems. Situation semantics was developed as an alternative informational semantics questioning many of the basic assumptions of the approach outlined above. Situation semantics is a relational semantics of partial worlds called situations. Sentences, instead of describing truth values, describe situations. Situations support (or fail to support) items of information, variously called states of affairs or infons. The partial nature of situations gives situation theory and situation semantics a flexible framework in which to model information and the context-dependent meaning of sentences in natural language. Situation theory is the mathematical ontology developed to give situation semantics a rigorous foundation. This thesis is intended to partially satisfy an as yet unmet need for a critical and comprehensive review of the situation theory literature.

Description of Situation Theory

Situation theory carries with it a rich ontology of objects. These include individuals, situations, relations, roles, infons, parameters, types and other abstracts, sets (both well-founded and non-well-founded), polarities and propositions. A basic infon σ is a unit of semantic content or information. It consists of a relation, an assignment mapping roles to objects, and a positive or negative polarity defining a set of logical duals. Complex infons are structural composites of atomic infons. An infon is not itself true or false, and so is not a proposition. In this way situation theory distinguishes itself from most languages of semantic information, which take sentences of a language like first-order predicate logic as propositions. A situation s is a part of the world that may make an infon *factual*. Typically situation theorists have had no use for, and an ontological aversion, to non-actual situations; hence a situation is often regarded as part of the actual world. If a situation s makes an item of information σ factual, we say that s *supports* σ , written $s \models \sigma$. Propositions are assertions that a situation supports an infon. The proposition that the situation s supports the infon σ is written $(s \models \sigma)$ ⁴. Because situations are not total worlds, a situation may support neither an infon nor its dual. In this way, the logic of situation theory resembles three-valued logics such as Kleene's strong three-valued logic (Restall 2006, 127), the generalizations of which have found numerous applications in database theory. Parameters are variable-like first-class objects that can fill the roles of a relation. An anchor is an assignment of objects to parameters, and the application of an anchor to a parameterized object replaces the parameters with the objects in the assignment. The parameters of a parametric object can be abstracted over to create

⁴ Sometimes the parentheses are omitted, blurring the distinction between statements and their associated propositions.

a rich variety of types and properties classifying the various objects of the theory. Such types are used as the basis for the development of the theory of information flow *between* situations underpinning the relational theory of meaning of situation semantics.

Roughly speaking, the meaning of natural-language utterances is taken to be a relation between types of utterance situations and types of situations described by the sentences. In order to model the semantics of natural language, situation theorists have sought to make their models as expressive as possible. In particular, they have attempted to maintain what Barwise has called an “open-door policy” to the theory (Barwise 1989n, 179-180): any meta-fact of situation theory should be expressible inside situation theory. As such, there are in general no restrictions on what kinds of situation theoretic objects can be constituents of infon. For example, situation theory permits an infon to fill a role in an infon’s relation. More shockingly, situation theory permits an infon to be a constituent of itself⁵. Thus, the objects of situation theory are not, in general, well-founded. For this purpose, a sophisticated theory of non-wellfounded structured objects had to be developed and adapted to the needs of situation theorists. This bold adoption of non-wellfounded models was remarkably successful, but was not without its critics (e.g., Grim and Mar 1989; Cargile 1990) or its difficulties. Despite their successes, situation theorists and semanticists did not find it easy to strike the appropriate balance between internal expressiveness and structural closure (Seligman and Moss 2011, 303).

⁵ Not all applications of situation theory require this expressivity, and so some models of the theory use only well-founded structures.

Overview of the Literature

Beginning with the work and collaboration of Jon Barwise and John Perry (1981, 1983, 1985), situation semantics and situation theory developed rapidly, and at times changed radically, over a more-than-decade long regime of vigorous collaboration among a diverse group of researchers. Much of the work in situation theory and situation semantics occurred under the auspices of the Situation Theory and Situation Semantics research group (STASS) at Stanford University's Center for the Study of Language and Information (CSLI). This group of researchers included Jon Barwise, John Perry, Robin Cooper, Keith Devlin, John Etchemendy, David Israel, Jerry Seligman and many others. The development of situation theory slowed substantially in the mid to late 1990s and especially after the death of Jon Barwise at the end of the last millennium. In addition to the loss of one of its most prominent proponents and creators, situation theory and situation semantics faced many conceptual and technical difficulties (Seligman and Moss 2011; Devlin 2004). Situation semantics' influence, while important, has been felt "more in terms of adoption of its broad themes than in terms of adoption of its specific formalism and proposals," (Perry 1998b, 671). Jon Barwise, himself, went on to work on related but more general issues of information flow in the late 1990s before his untimely passing. Much of this work, a body of mathematics called channel theory, was done in collaboration with Jeremy Seligman and is an outgrowth of Seligman's PhD thesis work (1990a) on a mathematical model of perspectives in situation theory. Their work together culminated in their book, *Information Flow: The logic of distributed systems* (Barwise and Seligman 1997). However, important work in situation theory and situation semantics continues, especially by Robin Cooper, Keith Devlin, Angelika Kratzer, and Jonathan Ginzburg.

A brief and non-technical introduction to situation theory and situation semantics can be found in Devlin (2006), and also in Perry (1998b). A thorough and highly technical analysis of situation theory may be found in Seligman and Moss (2011). Devlin (1991a) is a thorough and respected introduction to situation theory and situation semantics. An extensive overview of many issues discussed in situation semantics may be found in Kratzer (2009). An excellent introduction and overview to Jon Barwise's many publications in situation semantics is given in Devlin (2004).

We may identify several major works in situation theory. Foremost among these is the seminal work in the development of situation semantics in Barwise and Perry (1981, 1983). In this work, the basic themes of situation theory were first expounded. Valuable reviews and criticisms of this work can be found in Volume 8 Issue 1 of the journal *Linguistics and Philosophy* published February 01, 1985, explicitly devoted to their book *Situations and Attitudes*. This collection includes Scott Soames' widely influential critique (1985, 1986) of situation semantics' approach to definite descriptions. Another useful review includes the constructive analysis in Lindström (1991).

Adopting Peter Aczel's work (Aczel 1988; see also Barwise and Moss 1996) in non-well-founded set theory, Barwise and Etchemendy (1989) introduce non-well-founded sets into situation theory in order to provide a solution to the liar paradox⁶, a substantive if controversial achievement. Cogent critiques of their solution may be found in Cargile (1990), Grim and Mar (1989), and Gupta (1989).

In 1989, Jon Barwise published a widely cited volume of his essay on topics in situation semantics and situation theory. In Barwise (1989i), he offers an

⁶ Briefly, the liar paradox is a semantic paradox involving the interpretation of sentences such as "This sentence is not true."

innovative analysis of the problem of common knowledge⁷, also using non-wellfounded set theory. A number of other essays in the same volume detail a number of technical aspects related to situation theory's use of non-wellfounded sets. These include Barwise (1989a, 1989e, 1989k, 1989n). This volume of essays includes a number of other valuable and widely cited essays not related specifically to non-wellfounded set theory or puzzles of self reference. These include his essay (1989f) outlining a series of questions that any model of situation theory must answer, and (1989m) in which Barwise introduces a model of a scheme of individuation⁸ for situation theory, and introduces a theory of perspectival situations.

A couple of years later Keith Devlin completed and published his well-received book *Logic and Information* (1991), an expansive but informal development of situation theory and situation semantics, and an in-depth look at situation theory as a practical tool of analysis. Following this work, Devlin in collaboration with Duska Rosenberg focused efforts at using these tools in the analysis of human interaction, and specifically in the analysis of cooperation in the workplace (1993, 1996, 2008).

Jean Gawron and Stanley Peter's (1990a) monograph on quantification and anaphora is a *tour de force* of situation semantics. Around this time David Israel and John Perry published two widely read papers (1990, 1991) exploring the nature of propositions in information reports and the flow of information in different kinds of information architectures. Many of these ideas were

⁷ In brief, if a true proposition P is common knowledge, then for any two agents a and b , a knows that b knows P , a knows that b knows that a knows that P , a knows that b knows that a knows that b knows that P , and on in infinite regress. This raises various thorny epistemological issues.

⁸ Roughly, a scheme of individuation picks out the entities in the world that become the objects of the theory. These include individuals, relations, and situations.

incorporated into later work on situation-theoretic work on constraints and information flow in situation theory and then, later, channel theory, e.g., Seligman (1990a, 1990b, 1991) and Barwise and Seligman (1994, 1997). Although it may be understood independently of situation theory, the channel theory of Jon Barwise and Jeremy Seligman, which in its most complete form appears in Barwise and Seligman (1997), may itself be viewed as an attempt to come to grips with the situation-theoretic notion of constraints, that is, the problem of how information flows between situations.

Barwise and Cooper (1991, 1993) attempt to systematize the results of situation theorists into a coherent and usable theory, and introduce a graphical notation for situation theory based on that used in discourse representation theory. A highly mature and systematic statement of formal situation theory is given in Seligman and Moss (1997), recently updated and republished in (2011)⁹; it has become a standard technical reference for many of the subsequent developments and applications of the theory. Their framework is highly modular and adaptable¹⁰. Perhaps the most notable achievement within Seligman and Moss's framework is the impressive analysis of interrogatives in Ginzburg and Sag (2001). Various recent works by Robin Cooper (2005a, 2005b), Jonathan Ginzburg (2005, 2010 in press), and Ginzburg and Cooper (2004) have adapted the situation theory framework to a Martin-Löf's Type theoretical framework supplemented with records. Ginzburg, Sag, and their proponents argue that a semantics built in the framework of type theory with records (TTR) can un-

⁹ We cite the updated version of their paper in this thesis.

¹⁰ In practice, some have found the framework of Seligman and Moss also to be somewhat cumbersome to use in particular applications (e.g., Ginzburg 2010 in press).

problematically incorporate most of the results of situation semantics¹¹, while adding a rich body of theory.

A great deal of work in situation theory and situation semantics may be found collected in the three volumes entitled *Situation theory and its applications* (1990-1993) and in the first volume of *Logic, language, and computation* (1996), all published by the Center for the Study of Language and Information as part of their series CSLI Lecture Notes.

Situation theory has been applied to many problem domains, most obviously to problems in philosophical linguistics for which it was originally developed. Other topics to which situation semantics has been applied include propositional attitudes (e.g., Barwise and Perry 1981, 1983, 1985; Devlin 1991a; Ginzburg 1993), structure of metaphor (e.g., Mori and Nakagawa 1991), problems of linguistic disambiguation and partial information games (Parikh 1990, 2007), and natural-language processing and others problems of computational linguistics (Rieger 1995), among many other topics of interest. Also a number of scholars have applied situation semantics to language-specific problems¹². These include Japanese honorifics (Sugimura 1986), Japanese grammar (Suzuki and Tutiya 1991), Iroquoian-linguistic perspectives (Zaefferer 1991), the semantics of Spanish past-tense verbs (Cipria and Craige 2000) and Turkish case markings (Kılıçaslan 2006).

Notable contemporary work in situation semantics include Angelika Kratzer's work on conditionals, counterfactuals, definite descriptions, and

¹¹ The major exception is non-well-founded phenomena.

¹² It is worth noting that most work in situation semantics on topics ostensibly of universal linguistic concern occurs in English, and is unmarked as such, while work devoted to analyses of bits of non-English language are usually marked as language-specific analyses. Ethnocentric modes of thought frequently manifest themselves in patterns of marked and unmarked categories.

interrogatives (1989, 1998, 2002, 2009)¹³, Paul Elbourne's work on donkey anaphora and definite descriptions (2005), Francois Recanati's work on discourse situations and descriptions (1996, 2004), Eytan Zweig's work on minimal situations and anaphora (Zweig 2006), Fernando's (2009, 2010) recent modeling of situations, and Loukanova's work (2001, 2002) on definite descriptions and generalized quantifiers in situation semantics. It should be noted that much of this work, e.g. that of Angelika Kratzer, differs in a number of technical ways from the mainstream of the situation semantics program initiated by Jon Barwise and John Perry.

Situation theory has been applied to a number of other problems too. These include applications to problems of diagrammatic reasoning (Shin 1991; Stenning and Oberlander 1991), modeling real-world human reasoning (Devlin 2009), cooperative action and information-systems design (Devlin and Rosenberg 1993, 1996, 2008), threat assessment (Steinberg 2009), and legal-reasoning systems (Tojo and Wong 1996). Li et al. (2009) apply situation-theoretic tools to the analysis of the chain-store paradox in rational-action theory. Many researchers have attempted to clarify situation theory's relationship to various non-standard logics. These include relevant logic and other paraconsistent logics (Restall 1996), (Mares 2008, 2010), event and process logic (Georgeff et al. 1993), and episodic logics (Hwang and Schubert 1993). Developing a viable logical calculus for situation theory has been a central challenge for its proponents. In addition, there have been several attempted (if partial) implementations of situation theory as Prolog-esque programming languages, most notably PRO-SIT (Nakashima et al. 1988), BABY-SIT (Tin and Akman 1995, 1996), and ASTL (Black 1992). Also

¹³ Kratzer's work departs in some important ways from the mainstream of situation semantics. However, a discussion of Kratzer's work is outside the scope of our thesis.

Alan Cooper has implemented a type-theoretic system incorporating many ideas from situation theory (Cooper 2008).

Some more recent work in computing applications include the formulation of formal benchmarks for XML retrieval (Blanke and Lalmas 2006), ubiquitous computing (Wang et.al. 2009), modeling of cooperating processes (Kim and Lee 2007), modeling of agent actions in P2P systems (Brzykcy and Bartoszek 2007), communication flows in audiovisual media (Aguilar et al. 2009), and ontologically based situation awareness (Kokar et al. 2009), (Baumgartner et al. 2010). However, recent applications of situation theory have been rather sparse, and it is not always clear that they have been especially successful or interesting.

Organization of Thesis

This thesis is divided into several sections. In our section entitled Situation Theory we give the reader a relatively complete but informal introduction to situation theory. In the following section entitled Situation Semantics, we give a brief introduction to situation semantic's analysis of the meaning of sentences in natural language. This section is necessary, but also necessarily brief. It is followed by a section entitled Information Flow, in which we give an in-depth overview of the situation-theoretic literature on information flow .

In each of these, no attempt is made at presenting a rigorously coherent picture of the theory. Instead we focus on reviewing the variety of proposals in the literature. We justify this in two ways. First, the situation-theory literature already includes attempts at systematic and complete syntheses of the theory, e.g. Devlin (1991), Seligman and Moss (1997; 2011), and Barwise and Cooper (1991; 1993). Secondly, there are many situation theories, not just one—none of which is generally considered canonical or completely satisfactory, and many of which rest

on different and sometimes incompatible assumptions¹⁴. As Jeremy Seligman and Lawrence Moss note (1997; 2011, 253-254), the goals of presenting a comprehensive overview of the different models of situation theory in the literature and presenting a consistent description of the theory are incompatible.

¹⁴ Jon Barwise (1989, 255-276) presents a list of branching points at which different versions of situation theory may diverge; the philosophical ramifications of decisions at each of these points may be profound and are not always so well understood (Seligman and Moss 2011, 314). Although some might find this circumstance cause for doubt about situation theory's prospects, Jon Barwise argues otherwise, suggesting that the undoubtedly healthy mathematical field of topology tolerates tremendous diversity in its models and assumptions (1989, 255-256).

SITUATION THEORY

Situation theory begins with a universe of objects of different sorts derived from a *scheme of individuation*. This scheme of individuation may be a subjective classification of the discriminable world of some agent, or a classification of the world according to some theory. Whatever its basis, situation theory usually begins with a scheme of individuation as a given.

Infons and the Space of Issues

In situation theory, the world is seen as determining—relative to a scheme of individuation—a space of issues that may be decided in one way or another by parts of that world deemed situations. For example, whether John filched an apple at the Farmer’s Market last Saturday might be an issue that can be decided by a situation, in particular by a situation at the Farmer’s Market on Saturday. We can discern a closely related issue to the one just described, namely the issue of whether John did *not* filch an apple at the Farmer’s Market last Saturday. These two issues are duals; each is a polar opposite of the other, and they cannot both be decided positively in a situation. That is, it is not possible that John both did and did not steal that apple at the Farmer’s Market last Saturday. However, it is quite possible for different parts of the world to fail to decide the issue either way. For example, the momentous situation in the Theatre of Pompey on the Ides of March in 44 BC would have nothing to say about John’s petty thievery.

Situation theory needs some way to define or describe these issues. It does so by means of a structured informational object called an *infor*, the fundamental unit of information in situation theory¹. Infons are “the objects which actual

¹ Infons are sometimes called states of affairs or SOAs in the literature, especially in early situation theory.

situations make factual...[serving] to characterize the intrinsic nature of the situation” (Barwise 1989f, 264). Devlin (1991a, 38) also calls them the basic units of information but describes them ultimately as “artifacts of a theory that enable us to proceed.” And Ginzburg and Sag (2001, 83) describe infons as “perform[ing] the function of designating properties that situations might possess,” warning that “far from being sentences in a formal language, [infons] are non-linguistic abstractions individuated in terms of real-world objects.”²

Situation theory distinguishes two fundamental categories of infons: *basic* infons and *complex* infons. The complex infons are structural composites inductively constructed from a collection of basic infons. We therefore postpone our discussion of the complex infons.

Basic Infons

Basic infons are also structural composites having three parts: a *relation*, an appropriate *assignment* of objects to that relation, and a *polarity*. We describe these below, before giving an informal definition of the basic infon.

Relations. Relations abstract away the various concrete arrangements between the individuated elements of the world to describe the informational uniformities of the world (Barwise 1989n, 180). Note that situation theory does not presume to identify a relation with its graph; in particular, situation theory is

² This means, among other things, that if in one depiction of an infon we have the Morning Star as a constituent and in another depiction we have the Evening Star as a constituent, these two depict the same infon, which in fact has the same object (the planet Venus) as its constituent (Ginzburg and Sag 2001, 83). In this way situation theory attempts to avoid some of the semantic puzzles of reference. Note, however, that (Bremer and Cohnitz 2004, 163) have argued this stance may, under certain circumstances, seem to commit situation theory to a radical modal realism regarding possibilities. We do not feel competent to answer this question, but it seems to us, however, that Bremer and Cohnitz’s argument rests on a somewhat dubious construal of an abstracted parameter in a situation type as a thing having a reference apart from any anchor.

compatible with there being intensional distinctions between extensionally identical relations (Seligman and Moss 2011, footnote 4). Roughly, a relation is a structured object consisting of its identity, a collection of *argument roles* and a collection of *appropriateness conditions* associated with each role (Devlin 1991a, 113-128)³. The collection of roles determines the arity of the relation. Generally it is assumed that a relation has a finite arity. The appropriateness conditions of a role determine whether a particular object may *fill* that role. This is done so to preclude certain nonsensical assignments of objects to roles. For example, the relation *Steal* arguably has at least the roles for the *thief* and the *thing stolen*, with certain natural restrictions on the sorts of things that can be a thief or a thing stolen⁴.

Assignments. An assignment function fills argument roles in the relation of an infon by mapping argument roles to appropriate objects from a universe determined by the scheme of individuation⁵. An assignment of an object to a role is appropriate if that object satisfies all the appropriateness conditions of that role. We say that an assignment function is appropriate if all of its assignments are appropriate. A basic infon is well-formed only if its assignment function is appropriate⁶.

³ Some authors include additional properties of a relation such as minimality conditions. These need not concern us at present.

⁴ In this way we can preclude from consideration whether or not the bitterness of an unripe pineapple stole the time complexity of the QuickSort algorithm from the 111th digit of Pi.

⁵ Note that, according to the open-door policy we have already described, objects mapped to roles may, if appropriate to the role, themselves be roles, assignment functions, relations, infons, and so on.

⁶ Note that appropriately filling the roles of relation is not the same thing as being factual. Both Leona and a particular apple are both appropriate assignments to the roles of thief and thing thieved respectively, but obviously this does not mean that Leona actually stole the apple. An infon is factual just in case it is supported by some situation.

Situation theory does not require that every role in a relation be filled. Infons with partial assignments are called *unsaturated infons*. An infon that is not unsaturated is called *saturated*. Unsaturated infons play an important part in the representation of partial information in situation semantics. For clarity of exposition, we postpone any further discussion of unsaturated infons.

Polarity. The polarity of a basic infon describes whether or not a situation's support of that infon is to be taken as indicating that the arguments stand in the infon's relation. A basic infon with positive polarity indicates the issue of whether the objects do stand in that relation. A basic infon with negative polarity indicates the issue of whether the objects do *not* stand in that relation. Two basic infons alike in all respects⁷ except their polarity are said to be *duals*.

Having described the three parts of a basic infon, we give the following informal definition of a basic infon.

Definition 1.1 Basic Infon. A basic infon $\sigma = \langle\langle R; \alpha; i \rangle\rangle$ is a structure consisting of an n -ary relation R , a partial assignment function $\alpha : \text{Rol} \rightarrow \text{Obj}$ mapping roles to objects, and a polarity $i \in \{+, -\}$. An infon $\langle\langle R; \alpha; + \rangle\rangle$ is said to have *positive polarity* and an infon $\langle\langle R; \alpha; - \rangle\rangle$ is said to have *negative polarity*⁸.

Remark. Although the use of an assignment function makes an ordering scheme unnecessary, it is often convenient to use a conventional ordering on an informal basis when there is no chance of being misunderstood. For example, if

⁷ What it means to be alike in all respects is less simple an issue as it might first appear. We will, however, postpone further discussion of the identity of infons until we have introduced more of the theory.

⁸ The polarity of infons has been represented in a number of ways in the literature. These include the symbol sets $\{+, -\}$, $\{1, 0\}$, and $\{\text{True}, \text{False}\}$. Some authors will omit an explicit indication of the polarity in the case of positive infons, or when the collection of infons is closed under the structural operation of negation.

the relation is the relation of being numerically *greater than* we might have an infon representing the information that the integer 9 is greater than the integer 2:

$$\langle\langle \geq; 9, 2; + \rangle\rangle.$$

Obviously, the roles in this relation should have appropriate conditions restricting assignments to an appropriate numerical domain.

When we merely want to emphasize the roles of the infon, we may use a functional notation:

$$\langle\langle R; \alpha(r_1), \dots, \alpha(r_n); i \rangle\rangle$$

in which r_1, \dots, r_n are the argument roles in the domain of the assignment function, which may or may not be all the roles associated with the relation R .

When we want to make both the roles of the assignments explicit, we may use a notation adopted from Devlin (1991a):

$$\langle\langle R; r_1 \rightsquigarrow \mathbf{o}_1, \dots, r_n \rightsquigarrow \mathbf{o}_n; i \rangle\rangle$$

in which r_1, \dots, r_n are the roles of R in the domain of definition of the assignment function α , and $\mathbf{o}_1, \dots, \mathbf{o}_n$ are the objects assigned to those roles by the assignment function α , i.e., $\mathbf{o}_i = \alpha(r_i)$ for each i .

Depictions of infons vs. infons. It is useful to stop here to remind the reader of situation theorists' realist bent, and to caution the reader to be careful to distinguish the objects of the theory (e.g., infons, parameters, situations, etc.), from their presentations in the language of situation theory (e.g., terms, expressions, etc.). The same item of information is presentable in more than one way (Gawron and Peters 1990a, 22; Ginzburg and Sag 2001, 83), and so one must

be careful to distinguish the objects of situation theory from their modes of presentation. For example, the following are merely *different depictions* of the same infon, since the morning star is the same individual as the evening star (Venus) and since Tully and Cicero are different names for the same individual:

⟨⟨*Admires*; **Tully, Morning Star**; +⟩⟩

⟨⟨*Admires*; **Cicero, Morning Star**; +⟩⟩

⟨⟨*Admires*; **Tully, Evening Star**; +⟩⟩

⟨⟨*Admires*; **Cicero, Evening Star**; +⟩⟩.

As a practical concern, it behooves one to avoid such ambiguities in the depictions chosen to represent items of information.

Situations

Situations are also first-class objects of the theory. Devlin describes situations as “part[s] of the activity of the world,” (1991a, 11) and as “highly structured...parts of the world that [an] agent’s behavior discriminates,” indicating that the situations of situation theory correspond more or less to what we mean by situations in every day life. Of situations Barwise (1989, xiv) writes, “‘situation’ is our name for those portions of reality that agents find themselves in, and about which they exchange information.” Gawron and Peters (1990a, 16) introduce situations as, “limited parts of the world containing individuals and other objects, having properties and standing in relations,” and Barwise (1989i, 205) suggests to his readers that they think of a situation as “representing a chunk of information in

terms of a set of basic facts, where a fact is simply some objects standing in some relation.” Situations may be static parts of the world or events with temporal scope. Ginzburg and Sag (2001, 83) think of situations as “partial, temporally located, actual entities, whose role is to explicate such objects as states or events.” For some, e.g. Barwise (1989n, 185), one can even have situations in abstract, mathematical, universes. Despite Sowa’s (2000, 285-288) observation that a fully adequate definition of a *meaningful* situation has not been forthcoming—and it is true that the best way to model a situation has been much debated—it is clear that what matters most about situations is that *they constitute partial informational contexts where the issues raised by infons may be (or may fail to be) decided positively, and that they act as sites for information flow about other situations via natural, nomic, and conventional constraints*. These two aspects of situations are kept distinct in situation theory, and constitute a distinguishing hall-mark of situation theory’s approach to information.

The Support Relation Between Situations and Infons

For each issue raised by an infon, and for each situation individuated by a scheme of individuation, that issue will either be decided, or it will not be decided. When a situation decides⁹ an issue raised by an infon, that situation is said to *support* that infon and that infon is said to be *factual*. Let s be a situation, σ be a basic infon¹⁰. If s supports σ , we write $s \models \sigma$. The relation \models is called the *supports*

⁹ Our use of the verb *decides* is somewhat non-standard in the logic literature in that when a situation supports an infon it can only decide the issue raised by that infon positively. Situations are partial. The failure of an actual situation to support an infon does not mean that that infon will be unsupported by every other actual situation, with the sole exception of the actual world (the actual situation deciding all issues). If the actual world fails to support an infon, then no other actual situation can support it.

¹⁰ A situation’s support of a complex infon is determined by that situation’s support of its constituent basic infons, but the exact conditions for this support are determined by the kind of structural composite that complex infon happens to be. We defer any further discussion of these until we come to our section on complex infons.

relation. The *Austinian proposition* that s supports σ , written $(s \models \sigma)$ is true iff $s \models \sigma$ ¹¹.

We designate by $\bar{\sigma}$ the *dual* (or negation) of the infon σ . If σ is a basic infon then its dual is the same in all respects except that it has the oppositely-valenced polarity. For complex infons, it is not so simple. Not all situation theorists hold that every complex infon has a dual—some theorists hold that the negation of a complex infon is not well-formed.

At the level of Austinian propositions, situation theorists will typically assume a classical logic. Because situations are partial, not every issue will be decided by any given situation. Among other things, this means that $(s \models \sigma) \vee (s \models \bar{\sigma})$ is not a necessarily-true proposition. This should not be surprising since infons are not the propositions of situation theory. In particular $(s \models \bar{\sigma})$ is not the negation of the proposition $(s \models \sigma)$. The proposition $\neg(s \models \sigma)$ is the negation of the proposition $(s \models \sigma)$. We will sometimes write $(s \not\models \sigma)$ instead of $\neg(s \models \sigma)$. Another way of stating this is that for any situation s and basic infon σ , it is possible that $\neg(s \models \sigma) \wedge \neg(s \models \bar{\sigma})$ is true. However, situation theorists have generally assumed that any actual situation is coherent, that is, for any actual situation s and basic infon σ , the proposition that $(s \models \sigma) \wedge (s \models \bar{\sigma})$ is necessarily false. Some situation theorists have made the stronger claim that if $(s \models \sigma)$ then $\neg(s' \models \bar{\sigma})$ for every situation s' in the universe of situations. As it turns out, this cannot be the case for unsaturated infons, and perhaps other infons as well.

We use an example to make this more concrete. Suppose that for some infon the issue at stake is whether Leona won a trophy for bowling a perfect game

¹¹ There are actually two sorts of Austinian propositions. We have described one kind. The second is between an object and a type. If b is an object and T is a type, then written $(b : T)$ is the proposition that b is of type T . We will discuss later.

on the 9th of May. That issue can only be decided one way or another by a situation in which Leona was bowling on the 9th of May. Presumably, neither a¹² situation at the Farmer's Market on the following day (the day in which John filched the apple) nor a situation in Rome in 44 BC can decide¹³ the issue of Leona's performance sweeping the nines on the 9th of May.

Parameters and Abstraction

Throughout early situation theory, the universe of situation theory included variable-like¹⁴ objects called *parameters*. Like most variables, parameters may be given (or replaced by) values. Parameters also serve as sites for abstraction, yielding situation theory a powerful means to form types, properties, and relations. Situation semanticists have made wide use of parameters, especially of what are called *restricted parameters*, in the analysis of natural-language semantics¹⁵.

Roughly, a parametric object is a structured object containing parameters in place of other 'ordinary' objects. The parameters in a parametric object are given values through the application of an *anchor* to the parametric object. An anchor is a partial function f from the domain of parameters to a co-domain of objects. The application of an anchor to a parametric object replaces the free parameters of the

¹² We use the indefinite article here because an item of information will be supported by many different situations, just as how multiple overlapping regions may contain the same individuals. The range of situations that support Leona's bowling performance or John's skillful thievery may also include situations that support both: for example, a large situation that includes both the bowling alley on the 9th of May and the Farmer's Market the next day might decide both the issue of Mary's bowling a perfect game and of John's filching the apple.

¹³ Nonetheless, it is possible that a situation in the Farmer's Market might carry the information that Leona bowled a perfect game. For example Leona might have met her friend Thomas at the market and shown him a picture of her prize. We will return to the important issue of information flow in the last section of the thesis.

¹⁴ Gawron and Peters (1990, 18) describe parameters as being models of variables.

¹⁵ Much about the meaning of pronouns can be modeled as restrictions on parameters (Gawron and Peters 1990, 11).

parametric object with the objects in the range of the anchor. The objects assigned by the anchor must meet any appropriateness conditions dictated by each argument role of the object. We call an anchor whose assignments are appropriate an appropriate anchor.

Depending on the sorts in the situation-theoretic universe being described, there will be a variety of parameterized objects. We will describe two of the most important: *parametric infons* and *parametric propositions*. Because it is useful to distinguish parameters from non-parameters within infons, we adopt the convention of representing parameters as italicized and dotted single letters and representing other objects featured in structured objects in bold-face. We do not bold-face situations, roles, or parameters, except when they appear as arguments in an infon's relation. It will not be possible to observe this convention in every instance.

Parametric Infons

The infons we have described so far are often called *states of affairs* or *non-parametric infons*. Situation theory also includes *parametric infons* in which at least one of the roles is labeled by a parameter instead of an ordinary object. We give the following informal definitions.

Definition 1.2. Let X be a set of parameters and let $\sigma = \langle\langle R; r_1 \rightsquigarrow \mathbf{o}_1, \dots, r_n \rightsquigarrow \mathbf{o}_n; i \rangle\rangle$ be a basic infon. A parameterization of σ by the parameters in X involves the replacement of objects $\mathbf{o}_i, \dots, \mathbf{o}_j \subseteq \mathbf{o}_m, \dots, \mathbf{o}_n$ in σ with parameters from X . The application of an appropriate anchor f to a parameterized infon $\sigma = \langle\langle R; r_1 \rightsquigarrow \mathbf{o}_1, \dots, r_j \rightsquigarrow \dot{x}_j, \dots, r_k \rightsquigarrow \dot{x}_k, \dots, r_n \rightsquigarrow \mathbf{o}_n; i \rangle\rangle$, written $\sigma[f]$, is the (possibly parametric) infon in which every free parameter \dot{x} occurring in σ and in the domain of f is replaced with the value given by $f(\dot{x})$.

Remark. A parameter may label more than one role. Naturally, for any set of parameters, there will be many different possible parameterizations. Also, there will of course be many possible appropriate anchors for any parametric infon.

Example 1.1. Consider the relation of *Holding* . In addition to any roles for time and place, which we will ignore here, this relation will have a role for the *holder* and a role for the *thing being held* . Suppose that we have the following infon formed from this relation:

$$\langle\langle \textit{Holding}; \textit{holder} \rightsquigarrow \mathbf{John}, \textit{held} \rightsquigarrow \mathbf{b}; + \rangle\rangle$$

where \mathbf{b} is a particular briefcase. We may parameterize this infon, for example, by introducing a parameter y for the thing being held:

$$\langle\langle \textit{Holding}; \textit{holder} \rightsquigarrow \mathbf{John}, \textit{held} \rightsquigarrow y; + \rangle\rangle.$$

It is common to omit explicit mention of the roles when the meaning is clear. We do so here:

$$\langle\langle \textit{Holding}; \mathbf{John}, y; + \rangle\rangle.$$

One possible anchor f for this parametric infon might assign an apple \mathbf{a} to the parameter y so that if we apply this anchor to the parametric infon above, we get:

$$\langle\langle \textit{Holding}; \mathbf{John}, y; + \rangle\rangle[f] = \langle\langle \textit{Holding}; \mathbf{John}, \mathbf{a}; + \rangle\rangle$$

Parametric Propositions

A parametric infon cannot, it is argued, be made factual by a situation, absent an application of a particular anchor, since a parametric infon is

‘incomplete’. Therefore, some have been reluctant to extend the supports relation to parametric infons. However, many situation theorists extend the support relation as follows:

Definition 1.3. If s is a situation and σ is a parametric infon then $s \models \sigma$ iff there exists an appropriate anchor f such that $s \models \sigma[f]$.

Remark. Note that an anchor’s assignments must satisfy the appropriateness conditions associated with each role of the infon σ .

Thus under this definition any situation supporting $\langle\langle \textit{Holding}; \mathbf{Jerry}, \mathbf{b}; + \rangle\rangle$ will also support $\langle\langle \textit{Holding}; \dot{x}, \dot{y}; + \rangle\rangle$ since we have an anchor assigning **Jerry** to \dot{x} and assigning the briefcase **b** to \dot{y} .

We may distinguish two (overlapping) sorts of parametric propositions. The first is of the sort we just described, where a situation supports a parametric infon. We give here another example of such a parametric proposition:

$$s \models \langle\langle \textit{HasExclusiveLockOn}; \dot{p}, \dot{r}; + \rangle\rangle$$

The second sort is where a parameter stands in the place of a situation supporting a (possibly parametric) infon. We give an example of such a parametric proposition below:

$$\dot{s} \models \langle\langle \textit{HasExclusiveLockOn}; \mathbf{p}_1, \mathbf{r}_3; + \rangle\rangle$$

Parametric propositions of this second sort are of particular interest since they are the basis for the construction of situation types through abstraction.

We now turn to the important operations of abstraction and application.

Abstraction

Situation theory's rich collection of properties, relations, types and predicates arise through an operation called *abstraction*. Abstraction (called absorption by some situation semanticists) is intimately linked with the notion of parameters and anchors. The treatment of abstraction in much of the situation-theory and situation-semantics literatures is informal and uncomplicated, and was adequate for many of the purposes to which situation semanticists put it. Nonetheless, a more mathematically grounded theory of parameters, abstraction, and application was clearly needed, and for some purposes sorely needed until the work of Peter Aczel, Rachel Lunnon and others gave situation theory and situation semantics a mathematically-grounded theory of abstraction. For our purposes not much more than an informal sketch of a theory of abstraction is required; indeed anything more might unnecessarily bog down our presentation with a host of unnecessary details. We will, however, briefly review these important developments in situation theory, and refer our readers to presentations much more thorough than our own can be.

Simple Abstraction and Application

Let o be a parametric object and let $X = \{\dot{x}_1, \dots, \dot{x}_n\}$ be a set of parameters. We will write $[X | o]$ to indicate the abstract $Abs(X, o)$ that results from abstraction over the parameters $\dot{x}_1, \dots, \dot{x}_n$ of X occurring in o . Typically, if the number of parameters is few, we will drop the set notation and just list the parameters individually. The parameters $\dot{x}_1, \dots, \dot{x}_n$ in the abstract $[\{\dot{x}_1, \dots, \dot{x}_n\} | o]$ are bound or absorbed by abstraction. Depending upon the precise model there may be constraints placed on the binding of parameters in an abstract. In particular if the parametric object contains parametrically-restricted parameters, then not

every abstraction will be well-formed (Gawron and Peters 1990a). We will return to this issue when we discuss restricted parameters.

Roughly, the parameters bound by an abstract become the argument roles of the type, property, or relation defined by the abstract. The appropriateness conditions for assignments to those roles are determined by the role-specific appropriateness conditions governing the parameters in the parametric object abstracted upon.

Definition 1.4. An abstract $[\{\dot{x}_1, \dots, \dot{x}_n\} | o]$ is *applied* to an assignment $\alpha : rol \rightarrow obj$, and is written $[\{\dot{x}_1, \dots, \dot{x}_n\} | o].\alpha$. An assignment $\{\dot{x}_1 \rightsquigarrow \mathbf{o}_1, \dots, \dot{x}_n \rightsquigarrow \mathbf{o}_n\}$ of objects to the roles $\{\dot{x}_1, \dots, \dot{x}_n\}$ of an abstract $[\{\dot{x}_1, \dots, \dot{x}_n\} | o]$ is appropriate if for each assignment $\dot{x}_i \rightsquigarrow \mathbf{o}_i$ in α , the object \mathbf{o}_i is appropriate for the role that the free parameter \dot{x}_i fills in the un-abstracted object o .

Remarks. We might rewrite this as: α is an appropriate assignment for an abstract $[\{\dot{x}_1, \dots, \dot{x}_n\} | o]$ if there is an appropriate anchor f such that $[\{\dot{x}_1, \dots, \dot{x}_n\} | o].\alpha = o[f]$.

We will sometimes abuse our notation and write

$$[\{\dot{x}_1, \dots, \dot{x}_n\} | o].\{\mathbf{o}_1, \dots, \mathbf{o}_n\}$$

to indicate the application of an abstract to an assignment, relying upon either subscripted indices or the relative positions of each abstracted parameter and object to indicate the assignment.

Parametric Abstracts

Note that there may be other parameters in o not in the set $\{\dot{x}_1, \dots, \dot{x}_n\}$ that will remain free in o . If not every free parameter in o is bound in an abstract, then

we call the abstract a *parametric abstract*. We can apply an anchor to a parametric abstract in the same way that we apply an anchor to a parametric infon to obtain a non-parametric abstract. Given a parametric abstract $[Y | o(\dots, \dot{x}_1, \dots, \dot{x}_n, \dots)]$ with free parameters $\dot{x}_1, \dots, \dot{x}_n$ and an appropriate anchor f , then¹⁶:

$$[Y | o(\dots, \dot{x}_1, \dots, \dot{x}_n, \dots)] [f] = [Y | o(\dots, f(\dot{x}_1), \dots, f(\dot{x}_n), \dots)].$$

Any further discussion is best done in the context of a discussion of two particular kinds of abstracts of particular importance to situation theory: *infon abstracts* called *relations* (or *properties* if unary), and *propositional abstracts* called *types*.

Infon Abstracts

An infon abstract is of the form $[X | \sigma]$ where X is the set of bound parameters and σ is a parametric infon. If X is a singleton set, then $[X | \sigma]$ is called a *property*. Otherwise it is called a *relation*. Note that not every property or relation need arise in this way. For example, relations may be primitives given by an agent's scheme of individuation (Devlin 1991a, 63). Given an appropriate assignment α , $[X | \sigma].\alpha$ is the infon which results from replacing each parameter in σ with its assignment. This is best illustrated using an example.

Example 1.2. Property. Let $\langle\langle R; \alpha(r_1), \dots, \dot{x}, \dots, \alpha(r_n); i \rangle\rangle$ be a parametric infon with free parameter \dot{x} . Let us abstract over the parameter \dot{x} to form the property

$$P = [\dot{x} | \langle\langle R; \alpha(r_1), \dots, \dot{x}, \dots, \alpha(r_n); i \rangle\rangle].$$

¹⁶ Realizing that the notation might be confusing, we must emphasize that in the above example an anchor is being applied to the parametric abstract rather than the abstract being applied to an assignment. The latter is indicated by the use of a dot between abstract and assignment.

Let $\{\mathbf{o}\}$ be an appropriate assignment (using our abused notation). Then

$$[\dot{x} | \langle\langle R; \alpha(r_1), \dots, \dot{x}, \dots, \alpha(r_n); i \rangle\rangle]. \{\mathbf{o}\} = \langle\langle R; \alpha(r_1), \dots, \mathbf{o}, \dots, \alpha(r_n); i \rangle\rangle.$$

Infons are neither true nor false; instead they must be supported by a situation to be true or false. Let us write $P(\mathbf{o})$ for $\langle\langle R; \alpha(r_1), \dots, \mathbf{o}, \dots, \alpha(r_n); i \rangle\rangle$, the infon stating that the object \mathbf{o} has property P . $P(\mathbf{o})$ is factual iff there exists a situation s supporting it iff there exists a situation such that

$$s \models [\dot{x} | \langle\langle R; \alpha(r_1), \dots, \dot{x}, \dots, \alpha(r_n); i \rangle\rangle]. \{\mathbf{o}\}.$$

Restated slightly, $P(\mathbf{o})$ is factual iff there is a situation s and appropriate anchor f such that

$$s \models \langle\langle R; \alpha(r_1), \dots, \dot{x}, \dots, \alpha(r_n); i \rangle\rangle [f],$$

where $f(\dot{x}) = \mathbf{o}$.

Example 1.3. Relation. Given the parametric infon

$$\langle\langle HasExclusiveLockOn; \dot{p}, \dot{r}; + \rangle\rangle,$$

we can create the relation of something (intended here to range over processes) having an exclusive lock on something (intended here to range over resources) by abstracting over the parameters \dot{p} and \dot{r} :

$$R = [\dot{p}, \dot{r} | \langle\langle HasExclusiveLockOn; \dot{p}, \dot{r}; + \rangle\rangle]$$

An application of this abstract to an assignment of the process \mathbf{p} and resource \mathbf{r} to \dot{p} and \dot{r} respectively yields:

$$[\dot{p}, \dot{r} \mid \langle\langle \text{HasExclusiveLockOn}; \dot{p}, \dot{r}; + \rangle\rangle]. \{ \dot{p} \rightsquigarrow \mathbf{p}, \dot{r} \rightsquigarrow \mathbf{r} \} = \langle\langle \text{HasExclusiveLockOn}; \mathbf{p}, \mathbf{r}; + \rangle\rangle$$

Note however that since R is a relation it can be the relation predicating over an assignment of an infon. For example:

$$\langle\langle [\dot{p}, \dot{r} \mid \langle\langle \text{HasExclusiveLockOn}; \dot{p}, \dot{r}; + \rangle\rangle]; \alpha; + \rangle\rangle$$

is an infon with R as its relation and with $\alpha = \{ \dot{p} \rightsquigarrow \mathbf{p}, \dot{r} \rightsquigarrow \mathbf{r} \}$ as its assignment (where \dot{p} and \dot{r} are understood not to be free parameters but the argument roles of the relation). This infon denotes the same thing as:

$$[\dot{p}, \dot{r} \mid \langle\langle \text{HasExclusiveLockOn}; \dot{p}, \dot{r}; + \rangle\rangle]. \alpha$$

and the same thing as:

$$\langle\langle \text{HasExclusiveLockOn}; \mathbf{p}, \mathbf{r}; + \rangle\rangle,$$

but is nonetheless structurally distinct from each of these. While this may seem an unnecessary complication, properties and relations arising out of abstraction give us a rich family of infons. For example, instead of abstracting over both parameters, we might abstract over the parameter \dot{r} to form the parametric property of being a resource exclusively held by some process. Or we might form the property of being a resource exclusively held by the process \mathbf{p} :

$$P = [\dot{r} \mid \langle\langle \text{HasExclusiveLockOn}; \mathbf{p}, \dot{r}; + \rangle\rangle]$$

and then form an infon using this property:

$$\langle\langle [\dot{r} \mid \langle\langle \text{HasExclusiveLockOn}; \mathbf{p}, \dot{r}; + \rangle\rangle]; \dot{r} \rightsquigarrow \mathbf{r}; + \rangle\rangle.$$

The second main type of abstract is the propositional abstract. Propositional abstracts may be used to form types. We turn to our discussion of propositional abstracts now.

Propositional Abstracts

Propositional abstracts are formed by abstracting over parametric propositions. Propositional abstracts form types of two sorts: *situation types* and *object types*, corresponding to the two types of parametric propositions we identified earlier. Again note that, like properties and relations, not all types need be derived through abstraction. Some may be primitive types individuated in the scheme of individuation. For example, propositions are naturally classified by the type PROPOSITION.

Given a parametric proposition p , we form a propositional abstract by abstracting over a collection of parameters X in p : $[X \mid p]$. We may apply this abstract to an appropriate assignment to obtain a proposition. Rather than go through the tedious process of re-describing the mechanisms of abstraction and application in the abstract, we will turn immediately to a discussion of situation types and object types.

Situation types. Situation types are higher-order uniformities over situations. The usual way in which situation types are formulated is by abstracting over a situation parameter in the supports role of the support relation¹⁷. If σ is an infon, then $T = [s \mid s \models \sigma]$ is the type of situation supporting σ . The infon σ is called the conditioning infon of the type, and is sometimes written $Cond(T)$. An

¹⁷ Note that while we usually denote a proposition by $(s \models \sigma)$, we may also denote the informational content of a proposition by the infon $\langle\langle \models; situation \rightsquigarrow s, infon \rightsquigarrow \sigma; + \rangle\rangle$, which if parameterized, may be abstracted in the usual way.

appropriate assignment to a situation type is a situation, although there may be further restrictions if \dot{s} is a restricted parameter. The application of this situation type to an appropriate assignment yields the expected proposition, i.e.

$$([\dot{s} \mid \dot{s} \models \sigma]. \{\dot{s} \rightsquigarrow s\}) = (s \models \sigma).$$

Given a situation type $T = [\dot{s} \mid \dot{s} \models \sigma]$, the *Austinian proposition*¹⁸ that a situation s is of type T is written $(s : T)$. For a situation type T , $(s : T)$ iff $(s \models \sigma)$.

If σ is a parametric infon, then $T = [\dot{s} \mid \dot{s} \models \sigma]$ is a parametric type. As expected from the definition of the supports relation for parametric infons, if a situation type T is parametric, then a situation s is of type T iff there exists some anchor f having an assignment for every free parameter in σ such that $s \models \sigma[f]$.

Example 1.4. We may be interested in the type of situation such that Jerry is holding his briefcase: $\beta = [\dot{s} \mid \dot{s} \models \langle\langle \text{Holds}; \mathbf{Jerry}, \mathbf{briefcase}; + \rangle\rangle]$. A situation s is of type β iff $s \models \langle\langle \text{Holds}; \mathbf{Jerry}, \mathbf{briefcase}; + \rangle\rangle$.

Example 1.5. The following situation type is the type of situation in which some person is holding a briefcase. This is a parametric situation type:

$$T = [\dot{s} \mid \dot{s} \models \langle\langle \text{Holds}; \dot{p}, \dot{b}; + \rangle\rangle].$$

In place of the individuals **Jerry** and **briefcase** we have restricted parameters $\dot{p}_{\langle\langle \text{isPerson}; \dot{p}; + \rangle\rangle}$ and $\dot{b}_{\langle\langle \text{isBriefcase}; \dot{b}; + \rangle\rangle}$. We will go into more detail about restricted parameters in short order, but for now, all we need to know is that these restricted

¹⁸ Thus we have two kinds of Austinian propositions, those involving types and the of-type relation, and those involving infons and the supports relation. (Devlin 1991, 63) calls the latter *infonic propositions*.

parameters guarantee that any appropriate anchor f maps \dot{p} and \dot{b} to individuals that are persons and briefcases respectively. Intuitively this is the type of situation in which some person is holding some briefcase. A situation s is of type T iff there exists some appropriate anchor f satisfying the restrictions on the parameters such that $s \models \langle\langle \text{Holds}; \dot{p}, \dot{b}; + \rangle\rangle[f]$.

Example 1.6. Let us define the type of situation in which a definite process \mathbf{p} has an exclusive lock on a definite resource \mathbf{r} :

$$[\dot{s} \mid \dot{s} \models \langle\langle \text{HasExclusiveLockOn}; \mathbf{p}, \mathbf{r}; + \rangle\rangle]$$

Any situation in which \mathbf{p} has exclusive access to \mathbf{r} will be of this type.

Object types. We may also define object types. An object type is a property or relation grounded by a situation. Given a situation s , a parametric infon σ and a collection of parameters X , $[X \mid s \models \sigma]$ is an object type. Object types have the general form:

$$T = [\dot{x}_j, \dots, \dot{x}_k \mid s \models \langle\langle R; \alpha(r_1), \dots, \dot{x}_j, \dots, \dot{x}_k, \dots, \alpha(r_n); i \rangle\rangle]$$

where s is the *grounding situation* of the type. A sequence of possibly repeating objects $\{\mathbf{o}_j, \dots, \mathbf{o}_k\}$ has type T , written $\{\mathbf{o}_j, \dots, \mathbf{o}_k\} : T$ iff there is an appropriate anchor f such that

$$s \models \langle\langle R; \alpha(r_1), \dots, \dot{x}_j, \dots, \dot{x}_k, \dots, \alpha(r_n); i \rangle\rangle[f],$$

where $f = \{\dot{x}_j \rightsquigarrow \mathbf{o}_j, \dots, \dot{x}_k \rightsquigarrow \mathbf{o}_k\}$.

Equivalently, we have:

$$\{\mathbf{o}_j, \dots, \mathbf{o}_k\} : T \text{ iff } s \models T.\{\mathbf{o}_j, \dots, \mathbf{o}_k\}.$$

Example 1.7. Let us define as a type of object a pair consisting of a process and a resource such that the process has an exclusive lock on that resource in a given situation s :

$$[\dot{p}, \dot{r} \mid s \models \langle\langle \text{HasExclusiveLockOn}; \dot{p}, \dot{r}; + \rangle\rangle]$$

A pair \mathbf{p}, \mathbf{r} will be of this type just in case

$$s \models \langle\langle \text{HasExclusiveLockOn}; \mathbf{p}, \mathbf{r}; + \rangle\rangle.$$

Remark. Note that if $s \not\models \langle\langle \text{HasExclusiveLockOn}; \mathbf{p}, \mathbf{r}; + \rangle\rangle$ then \mathbf{p}, \mathbf{r} would not be of this type even if in some other situation s' , $s' \models \langle\langle \text{HasExclusiveLockOn}; \mathbf{p}, \mathbf{r}; + \rangle\rangle$. Types are specific to situations.

Restricted Parameters

At its simplest, a *restricted parameter* is a parameter conditioned upon a set of infons being made factual. Let us define $\models \sigma$ to mean that the infon σ is factual, i.e., that there exists some actual situation s such that $s \models \sigma$. In the framework of Gawron and Peters (1990), we may informally define a restricted parameter as follows.

Definition 1.5 (Gawron and Peters 1990a). Let \dot{x}_C be the restriction of the parameter \dot{x} by a set of parametric infons C . An anchor f on \dot{x}_C is well-defined iff for each $\sigma \in C$, $\models \sigma[f]$ (and f anchors \dot{x}).

Remark. An important feature of this definition is that the conditioning infons of a restricted parameter may be supported by resource situations distinct from the situation supporting the principal infon.

Example 1.8. Let $\dot{x}_{\langle\langle isPerson; \dot{x}; + \rangle\rangle}$ be the restriction of the parameter \dot{x} by the parametric infon $\langle\langle isPerson; \dot{x}; + \rangle\rangle$ and let $\dot{y}_{\langle\langle isBriefcase; \dot{y}; + \rangle\rangle}$ be the restriction of the parameter \dot{y} by the parametric infon $\langle\langle isBriefcase; \dot{y}; + \rangle\rangle$. Then for any given situation s :

$$s \models \langle\langle Holding; \dot{x}_{\langle\langle isPerson; \dot{x}; + \rangle\rangle}, \dot{y}_{\langle\langle isBriefcase; \dot{y}; + \rangle\rangle}; + \rangle\rangle$$

iff there exists an anchor f assigning some \mathbf{p} to \dot{x} and some \mathbf{b} to \dot{y} and there exist situations s' and s'' such that

$$s' \models \langle\langle isPerson; \mathbf{p}; + \rangle\rangle,$$

$$s'' \models \langle\langle isBriefcase; \mathbf{b}; + \rangle\rangle,$$

and

$$s \models \langle\langle Holding; \mathbf{p}, \mathbf{b}; + \rangle\rangle.$$

Abstraction over restricted parameters is permitted. However unrestricted abstraction over restricted parameters, as they have been defined above, leads to inconsistency (Gawron and Peters 1990a, 93, 176-178). Therefore Gawron and Peters (1990a, 93) require that abstraction obey their Absorption Principle.

Let us say that if \dot{x} is a parameter in the restriction of a parameter \dot{y} then \dot{y} depends on \dot{x} (Gawron and Peters 1990a, 93). The Absorption Principle states that if $\dot{y}_{\langle\langle \dots \dot{x} \dots \rangle\rangle}$ is a restricted parameter occurring in an object o , then any abstraction on the object o cannot abstract over \dot{x} without also abstracting over \dot{y} . Thus the Absorption Principle precludes abstracts of the form

$$[\dot{x} \mid \langle\langle \dots \dot{y}_{\langle\langle \dots \dot{x} \dots \rangle\rangle} \dots \rangle\rangle]$$

because, it is argued, any anchor to the free parameter y would have no sensible interpretation.

Discussion

Much of the use of parameters and abstraction in early channel theory is remarkably informal (Westerståhl 1990, 1993), mostly because a formal mathematical theory of parameters and abstraction adequate to the expressive needs of situation theory had not yet been developed and because these tools proved too useful to situation semanticists to be abandoned. Nonetheless, situation theorists were painfully aware of the necessity of developing a rigorous mathematical foundation for situation theory if it were to achieve the ambitions of its proponents.

Developments in a theory of parameters and abstraction. Therefore, over the course of several years a rigorous mathematical theory of parameters, parametric objects, and abstraction, expressive enough to account for non-wellfounded objects (Aczel 1988), was successfully developed by Peter Aczel, Rachel Lunnon, and others. Aczel (1990) introduces a general theory of ontology and replacement system in order to come to an adequate and general definition of a substitution operation for structured objects, well-founded and anti-founded, such as those found in situation theory.

Briefly, a replacement system \mathcal{M} consists of three parts: a universe, a component function, and a replacement operation. The universe of \mathcal{M} consists of a class of structured objects M , where each object in M is either an atom or is composed of other objects from M . The component function $C : M \rightarrow \text{pow}(M)$ simply gives the set of components of each object $a \in M$, and the replacement

operation \bullet is defined¹⁹ such that for each function $\sigma : C(a) \rightarrow M$ there is an object $\sigma \bullet a$ in M obtained by replacing every component of a with that assigned by σ , subject to certain natural conditions.

However, the structured universes of Aczel (1990) do not include an explicit notion of parameter or parameter binding. Utilizing Aczel's theory of replacement systems, Westerståhl (1990) was among the first to attempt a rigorous definition of parameters and abstraction and parametric objects by the presentation of an internally consistent first-order theory of parameters. Given a replacement system \mathcal{M} and a class of parameters X , and operation of abstraction, Westerståhl expands M to $M[X]^*$, which consists of all the objects of M and the parameterizations of objects in M by X , $M[X]$, further closed under abstraction.

Aczel and Lunnon (1991) also formulate universes of structured objects with parameters for which they find a well-defined substitution operation, in both well-founded and anti-founded universes. Aczel and Lunnon (1991) define a λ -universe consisting of a universe having a proper class of parameters and a substitution operation, but which also includes a simultaneous abstraction operation and a class of lambda-abstracts, with identity of abstracts determined in terms of alpha-convertibility. They show that well-founded and anti-founded lambda-universes uniquely exist for any ontology. Other important formal foundational developments of the theory can be found in Fernando (1990), Lunnon (1991a), and Lunnon (1991b). Not satisfied with the complexity of the algebraic

¹⁹ Note that in the context of situation theory, the replacement operation is not sufficient for situation theory's needs for a substitution operation since replacement does not preserve appropriateness (Westerståhl 1990, 195).

approach taken in Aczel's previous work, Aczel (1996) presents a set theoretical meta-theory intended to be intuitive enough for informal use²⁰.

A few comments are in order about the abstraction operation introduced by Aczel and Lunnon. The abstraction operation is a generalization of the abstraction operation of the λ -calculus. Rather than being sequential over a set of parameters, parameters are abstracted simultaneously. For this reason, abstraction and application are technically defined over an indexed set of parameters and objects, rather than over the parameters and objects themselves. Given an anchor $\hat{f} : Param \rightarrow Objects$, a one-to-one index function $F : Indices \rightarrow Param$ and a one-to-one index function $f : Indices \rightarrow Objects$, assignments satisfy $\hat{f}(\dot{x}) = f(F^{-1}(\dot{x}))$ so that f is an appropriate assignment to an abstract $\lambda F.o$ iff $o[\hat{f}]$ is defined (Barwise and Cooper 1991).

Example 1.9. Given the parametric infon $\langle\langle HasExclusiveLockOn; \dot{p}, \dot{r}; + \rangle\rangle$, and the indexed set $[1 \mapsto \dot{p}, 2 \mapsto \dot{r}]$, we form the abstract

$$\lambda[1 \mapsto \dot{p}, 2 \mapsto \dot{r}].\langle\langle HasExclusiveLockOn; \dot{p}, \dot{r}; + \rangle\rangle.$$

The application of this abstract to the indexed set of objects $[1 \mapsto \mathbf{p}_1, 2 \mapsto \mathbf{r}_3]$ gives us the infon:

$$\langle\langle HasExclusiveLockOn; \dot{p} \rightsquigarrow \mathbf{p}_1, \dot{r} \rightsquigarrow \mathbf{r}_3; + \rangle\rangle$$

i.e.,

$$\langle\langle HasExclusiveLockOn; \mathbf{p}_1, \mathbf{r}_3; + \rangle\rangle.$$

²⁰ We judge that the approach is indeed more accessible than that given in his previous papers. We urge those readers interested in learning more about the mathematical basis of a theory of parameters and abstraction to consult this paper.

Barwise and Cooper (1991, 1993)²¹ utilize the lambda-universes of Aczel and Lunnon (1991) in giving an influential guiding statement for simple well-founded situation-theoretic universes²². Seligman and Moss (1997; 2011) and Ginzburg and Sag (2001) also make use of Aczel and Lunnon’s foundational work. However, in the models of situation theory they describe, the universe does not include parametric objects. Instead, abstraction on an object involves replacing arguments of that object with internal placeholders²³. For example, abstracting over \mathbf{p}_1 in the infon

$$\langle\langle HasExclusiveLockOn; \mathbf{p}_1, \mathbf{r}_3; + \rangle\rangle$$

we would get the abstract:

$$\lambda\{\mathbf{p}_1 \mapsto \pi\}.\langle\langle HasExclusiveLockOn; \mathbf{p}_1, \mathbf{r}_3; + \rangle\rangle$$

and assigning this abstract to the assignment:

$$f : \pi \mapsto \mathbf{p}_3$$

gives us back the infon:

$$\langle\langle HasExclusiveLockOn; \mathbf{p}_3, \mathbf{r}_3; + \rangle\rangle.$$

²¹ The purpose of their paper was both to summarize the current state of situation theory and to introduce a new diagrammatic notation for situation theory, Extended Kamp Notation, based on Kamp’s notation for Discourse Representation theory.

²² Cavedon (1995) uses Barwise and Cooper’s (1991) model of parametric objects and types in extending Jon Barwise and Jeremy Seligman’s theory of classifications to include parametric types.

²³ These objects are called pointers in Seligman and Moss (1997; 2011) and called placeholders in Ginzburg and Sag (2001).

Such an approach has the benefit of substantially simplifying the situation-theoretic universe. The sophisticated analysis of interrogatives in Ginzburg and Sag (2001) also indicates that the immediate flexibility sacrificed with the loss of parameters is not insurmountable. Moreover it answers some of the ontological concerns regarding parameters discussed in the next section.

Ontological status of parameters. The ontological status of parameters in the theory is an unusual one. Parameters have features unlike that found in the typical treatment of variables in logical and programming languages. As first-class objects, situation theory permits quantification over parameters; infons may even describe parameters as objects with properties. This has caused some to wonder whether the inclusion of parameters as first-class objects leads to inconsistencies. For example, it was not immediately apparent to situation theorists how an infon like:

$$\langle\langle isParameter; \dot{x}; + \rangle\rangle$$

can be *about* the parameter \dot{x} , and *not* be just a parametric infon waiting for an anchor (Westerståhl 1990, 194). Somewhat surprisingly, Westerståhl (1990) is able to show that this problem is only illusory and that the dual role of parameters as objects-to-be-talked-about and as variable-like entities taking values does not lead to inconsistency. In the process, Westerståhl urges us not to hold the analogy to propositional variables too close to heart:

It is misleading to think of a parametric proposition as analogous to an open formula $\phi(x)$. The proper analogy is with a *sentence* $\phi(c)$...Syntactic abstraction with variable-binding and object abstraction in a model are only analogous up to a point...The novel aspect is that the *same* object is used for two distinct, and in fact *unrelated*, purposes. (Westerståhl 1990, 198).

But there are problems of a more general kind too. For one thing, we do not necessarily expect parameters to be the sorts of things that would be individuated by an agent's scheme of individuation (Devlin 2006, 606). In some ways, parametric objects appear as less real than the situation-theoretic objects from which they are parameterized (Westerståhl 1990, 195). If they are merely conveniences afforded to the theorist, why should they be first-class objects²⁴? Such concerns have given cause for some situation theorists to seek out means by which the role of parameters in the theory might be removed or made more modest without thereby crippling its applicability to problems of natural language semantics (Ginzburg 1993, 274). For example, Crimmins (1993) investigates whether parameters are necessary at all to situation theory, and we have already discussed the models of Seligman and Moss (1997; 2011) and Ginzburg and Sag (2001).

Problems with restricted parameters. As useful as situation semanticists have found restricted parameters to be, working out some of their technical details has proved somewhat laborious. For one thing, as we discussed earlier in relation to the absorption principle of Gawron and Peters (1990a), the conditioning infons of the restricted parameter might themselves be parametric, raising certain technical issues. Also, other certain technical difficulties have been noted with parametric conditions on parameters (Westerståhl 1990). Even among those for whom parameters are an important part of situation theory, there are doubts about whether restricted parameters are necessary, or even particularly useful (e.g. Aczel 1996).

²⁴ See Westerståhl (1990) for a detailed defense of situation theory's use of parameters as first-class objects.

Barwise and Cooper (1991; 1993), acting upon the advice of Peter Aczel and Gordon Plotkin, among others, generalizes the notion of restriction to include arbitrary objects. Its may be given the general form of²⁵:

$$o \upharpoonright P$$

where o is an arbitrary object or proposition and P is a proposition restricting o .

The restriction operation is required to meet the following four conditions

(Barwise and Cooper 1991, 35):

1. $o \upharpoonright P$ is well-defined iff P is not false.
2. If P is true (and therefore not parametric) then $o \upharpoonright P = o$.
3. The \upharpoonright operation distributes over closure operators not abstracting over or substituting for parameters, and
4. $[X \mid o] \upharpoonright P = [X \mid o \upharpoonright P]$ given that no parameter in the proposition P is in X .

Example 1.10. For example, the parametric proposition

$$(s \models \langle\langle HasExclusiveLockOn; \dot{p}, \mathbf{r}; + \rangle\rangle)$$

might be restricted to the proposition:

$$(s \models \langle\langle HasExclusiveLockOn; \dot{p}, \mathbf{r}; + \rangle\rangle) \upharpoonright (s \models \langle\langle isProcess; \dot{p}; + \rangle\rangle).$$

However, non-parametric objects can be restricted as well. For example,

²⁵ Barwise and Cooper use a different, diagrammatic notation. We may elect to call \upharpoonright the *Plotkin Restriction Operator*, after Gordon Plotkin who originally suggested its use (Plotkin 1990).

$(s \models \langle\langle \text{HasExclusiveLockOn}; \mathbf{p}, \mathbf{r}; + \rangle\rangle) \uparrow (s \models \langle\langle \text{TimeStampAfter}; \mathbf{p}, 1101009130503020; + \rangle\rangle)$

is also permitted.

Seligman and Moss (1997; 2011) and Ginzburg and Sag (2001) also adopt variants of the general restriction operation of Barwise and Cooper (1991; 1993). In this regard, one must consider that the frameworks of Seligman and Moss (1997; 2011) and Ginzburg and Sag (2001) are more-or-less parameter-free, except as ‘placeholders’ internal to abstracts, as we have already stated.

Westerståhl, Haglund and Lager (1993) attempt a reconciliation of an approach similar to Barwise and Cooper (1993) and the approach of Gawron and Peters (1990a) by defining a formal language whose syntax observes the Absorption Principle of Gawron and Peters (1990a, 93) but that is interpretable in frameworks such as that of Barwise and Cooper (1993).

Having completed our discussion of parameters and abstraction, we begin our discussion of the complex infons.

Complex Infons

The complex infons are structural composites of basic infons. These structural composites may include conjunction, disjunction, and negation, and various sorts of quantified infons. Before continuing, we warn our reader that not all situation theorists admit all of these into their models of situation theory. For example, many situation theorists do not freely negate arbitrary infons, especially if quantification is somehow involved; as it turns out, quantification and negation of infons raise thorny philosophical and technical issues for the theory²⁶.

²⁶ In contrast, quantification and negation of propositions is generally taken to be unproblematically classical.

Conjunctions, Disjunctions, and
Negations of Basic Infons

Conjunctions, disjunctions, and negations of basic infons are not particularly controversial.

Conjunctions and disjunctions. One may construct a class of conjunctions and disjunctions of infons in the fairly standard way. Given two infons, σ and τ , either complex or basic, we can join them together into a conjunction $\sigma \wedge \tau$ or a disjunction $\sigma \vee \tau$. For any given conjunction or disjunction, each component infon will be either basic or complex. Complex infons can themselves be eventually decomposed into basic infons.

Situation theorists have frequently asserted that that for any situation s :

$$s \models (\sigma \wedge \tau) \text{ iff } s \models \sigma \text{ and } s \models \tau$$

and

$$s \models (\sigma \vee \tau) \text{ iff } s \models \sigma \text{ or } s \models \tau.$$

Negation of basic infons. The negation of basic infons is fairly straightforward and uncontroversial. Given a basic infon, its negation is simply its dual:

$$\overline{\langle\langle R; \mathbf{a}; + \rangle\rangle} = \langle\langle R; \mathbf{a}; - \rangle\rangle$$

and

$$\overline{\langle\langle R; \mathbf{a}; - \rangle\rangle} = \langle\langle R; \mathbf{a}; + \rangle\rangle.$$

DeMorgan's rules. Furthermore, negation of basic infons is frequently extended to negations of conjunctions and disjunctions of basic infons (or infons that are conjunctions, disjunctions, or negations of basic infons) according to DeMorgan's rules:

$$v = \sigma \wedge \tau \text{ iff } \bar{v} = \bar{\sigma} \vee \bar{\tau}$$

and

$$v = \sigma \vee \tau \text{ iff } \bar{v} = \bar{\sigma} \wedge \bar{\tau}$$

so that

$$s \models \overline{\sigma \wedge \tau} \text{ iff } s \models \bar{\sigma} \text{ or } s \models \bar{\tau}$$

and

$$s \models \overline{\sigma \vee \tau} \text{ iff } s \models \bar{\sigma} \text{ and } s \models \bar{\tau}.$$

We supplement this mainly uninteresting discussion with a brief look at Edwin Mares' (1999) theory of *unresolved disjunctions*.

Unresolved disjunctive information. In most versions of situation theory, a situation's support of basic infons is primary, and its support of complex infons secondary. Thus we have $s \models (\sigma \vee \tau)$ iff $s \models \sigma$ or $s \models \tau$. However, Edwin Mares (1999) proposes that in some cases situations support unresolved disjunctions, i.e. where situations support a disjunction without either supporting either of the disjuncts. In his paper Mares gives several examples of unresolved disjunctions. His first example draws upon the indeterminacy of the oscillation of photons passing through a polarizer, under a no-hidden-variables interpretation of quantum

physics. His second example is of a deterministic system of switches in which a necessary condition for a light's being on is that both switches be on, but where in a given situation there is only information about one of the switch's states. Mares develops a forking semantics akin to that found in temporal logics applicable to relevance logic. His semantics assumes both a partial order on situations and that the information supported by a situation is persistent: if it is supported by one situation then it is supported by any situation extending it in the partial order²⁷. The basic idea is that an unresolved disjunction is supported by a situation s iff for every $\delta \in D(s)$ there is some $s' \in \delta$ such that one of the disjuncts is supported by s' , where $D(s)$ is the set of all possible extensions of s and where each extension of s is a set of situations. His semantics depart in a number of interesting ways from the norm within situation theory (such as by introducing a four-valued semantics he attributes to J. Michael Dunn). However, deeper discussion of this work is beyond the scope of this thesis.

Mares' examples of unresolved disjunction appear to us to be clearer examples of information flow between situations than examples of information supported by situations²⁸. Mares, acknowledges that such an analysis is possible:

John Barwise has suggested that I could avoid the problem of unresolved disjunctions by distinguishing between the information *contained* in a situation from the information *carried* by that situation. Given this distinction, we could say that s does not contain the unresolved disjunctive information, but rather carries it by means of the laws of nature. In channel theory...this distinction is made rather nicely by distinguishing between sites and channels. (Mares 1999, footnote 2).

²⁷ We will soon discuss persistence in detail.

²⁸ We will discuss information flow in more details in our section of that name.

Quantification

There have been a number of proposals throughout the situation-theory literature on how to model quantification appropriately. Quantifiers may be modeled as axiomatic second-order types of the theory, or they may be constructed using the resources abstraction provides. In the latter case, one may be free to form a rich variety of generalized quantifiers. Two simple proposals of the first kind are those of Barwise (1989m) and Devlin (1991a). We also discuss how quantification is handled by Barwise and Cooper (1991).

Quantification in Barwise (1989m). In Barwise (1989m), quantified infons $\exists \vec{x}_\tau \sigma$ and $\forall \vec{x}_\tau \sigma$ are composed of either an existential quantifier \exists or a universal quantifier \forall , a set of restricted parameters \vec{x} conditioned on some infon τ , and a parametric infon σ with free parameter(s) \vec{x} such that:

$$s \models \exists \vec{x}_\tau \sigma \text{ iff there is an anchor } f \text{ mapping each } \dot{x} \text{ in } \vec{x} \text{ to some } \mathbf{b} \text{ for which}$$

$$s \models \sigma[f] \text{ and } s \models \tau[f].$$

and

$$s \models \forall \vec{x}_\tau \sigma \text{ iff for every anchor } f \text{ mapping each } \dot{x} \text{ in } \vec{x} \text{ to some } \mathbf{b} \text{ for which}$$

$$s \models \sigma[f] \text{ and } s \models \tau[f].$$

Barwise proposes that each quantified infon be given the expected duals²⁹:

$$s \models \neg(\exists \vec{x}_\tau \sigma) \text{ iff } s \models \forall \vec{x}_\tau \bar{\sigma}$$

and

²⁹ Doing so, Barwise abandons the idea that all infons must have the property of upwards persistence, a property much discussed later in this thesis.

$$s \models \neg(\forall \vec{x}_\tau \sigma) \text{ iff } s \models \exists \vec{x}_\tau \bar{\sigma}.$$

Example 1.11. Let us write the infon that indicates the item of information that there exists a **b** such that **b** is a briefcase and Jerry is holding it:

$$\exists \dot{b}_{\langle\langle isBriefcase; \dot{b}; + \rangle\rangle} \langle\langle Holding; \mathbf{Jerry}, \dot{b}; + \rangle\rangle$$

Thus, for any situation s ,

$$s \models \exists \dot{b}_{\langle\langle isBriefcase; \dot{b}; + \rangle\rangle} \langle\langle Holding; \mathbf{Jerry}, \dot{b}; + \rangle\rangle$$

if and only if there is an anchor f substituting some appropriate **b** for \dot{b} for which

$$s \models \langle\langle isBriefcase; \dot{b}; + \rangle\rangle[f] \text{ and } s \models \langle\langle Holding; \mathbf{Jerry}, \dot{b}; + \rangle\rangle[f].$$

Quantification in Devlin (1991a). Keith Devlin (1991a, 134-136) proposes that we define basic existential and universal quantifiers as follows:

$$s \models (\exists \dot{x} \in U) \sigma \text{ iff } s \models \sigma[f] \text{ for some anchor } f$$

mapping the restricted parameter \dot{x} to some element of U

and

$$s \models (\forall \dot{x} \in U) \sigma \text{ iff } s \models \sigma[f] \text{ for every anchor } f$$

mapping the restricted parameter \dot{x} to some element of U .

The proposal of Devlin (1991a 134-136) is very similar to the proposal of Barwise (1989m). There are three notable differences.

First, the restricted parameter need not be supported by the *same* situation as the situation being described by the quantified infon³⁰. The use of a (possibly) distinct *resource situation* to support the infon conditioning the restricted parameter affords us an indispensable additional degree of freedom³¹.

Devlin's proposal also differs from that of Barwise (1989m) in that quantification is explicitly *bounded* by some set U so that the parameter can only be mapped to an element of U by the anchor f . One advantage is that quantified infons are persistent, that is, if a situation supports them, then any larger situation extending it will support them.

Thirdly, unlike Barwise (1989m), Devlin does not countenance a negation operation on either basic or complex infons (including quantified infons) as such. Somewhat confusingly, Devlin (1991a, 267) *does* define the duals of quantified infons as follows:

$$\text{If } \sigma = (\forall \dot{x} \in U)\tau \text{ then } \bar{\sigma} = (\exists \dot{x} \in U)\bar{\tau} .$$

$$\text{If } \sigma = (\exists \dot{x} \in U)\tau \text{ then } \bar{\sigma} = (\forall \dot{x} \in U)\bar{\tau} .$$

³⁰ Devlin's notation does not explicitly indicate the conditioning infons of the restricted parameter. However, Devlin notes that for there to be an anchor f assigning an object from U to the parameter, the conditioning infons of the restricted parameter must be supported by some situation r , possibly distinct from the described situation s .

³¹ The utility of using a distinct resource situation is easily motivated. For example, if everyone who got an A on their exam is attending the convocation, then we may expect that the situation making factual that a person got an A on their exam need not be the same situation supporting their attendance at the convocation. In some cases it is arguably necessary that they be distinct (Cooper 1995; Gawron and Peters 1990a; Kratzer 2009).

Resource situations were introduced in Barwise and Perry (1983). We are somewhat surprised therefore to find that Barwise (1989m) does not use them here. It is true that some of the more substantial work on quantification in situation theory and situation semantics came a few years later. It is possible that Barwise did not wish to complicate his discussion of quantification, negation, and persistence. However, the use of resource situations to restrict the range of the quantifier can preserve the property of infon persistence.

Thus, the dual of $(\forall \dot{x} \in U)\langle\langle R; \alpha; - \rangle\rangle$ would be $(\exists \dot{x} \in U)\langle\langle R; \alpha; + \rangle\rangle$, and $s \models (\forall \dot{x} \in U)\langle\langle R; \alpha; - \rangle\rangle$ if and only if $s \not\models (\exists \dot{x} \in U)\langle\langle R; \alpha; + \rangle\rangle$ ³², given the principle that a situation cannot support both an infon and its dual.

Example 1.12. We adopt Barwise's convenient notation indicating the infons conditioning the parameters in order to adapt our previous example to reflect Devlin's proposal:

$$s \models (\exists \dot{b}_{\langle\langle isBriefcase; \dot{b}; + \rangle\rangle} \in U)\langle\langle Holding; \mathbf{Jerry}, \dot{b}; + \rangle\rangle$$

iff there is an anchor f substituting some appropriate \mathbf{b} in U for \dot{b} such that:

$$r \models \langle\langle isBriefcase; \dot{b}; + \rangle\rangle[f] \text{ and } s \models \langle\langle Holding; \mathbf{Jerry}, \dot{b}; + \rangle\rangle[f].$$

Quantification in Barwise and Cooper (1991). As is well known, quantifiers are higher-order relations or types. Barwise and Cooper (1991) introduce two forms of quantification, one in which quantifiers are modeled using types and the other where quantifiers are modeled as properties or relations. Quantifiers modeled as types range over the entire situation theory universe: $T : \exists$ whenever there is an appropriate assignment of objects satisfying T and $T : \forall$ whenever every appropriate assignment to T is of type T .

³² Devlin argues that the appropriate use of existentially and universally quantified infons involves there being sufficient information to decide the matter between the quantified infon and its dual: "in order for a negative utterance to be informational (in the intended manner), the speaker should ensure that the described situation is adequately identified... [A] cooperative use of a negative utterance...places on the speaker an obligation to ensure that the described situation as understood by the listener...is sufficiently rich to *decide* the relevant issue...one way or the other," (Devlin 1991, 265). Thus, Devlin argues that an utterance sufficiently rich to have informational contents like $s \not\models (\exists \dot{x} \in U)\langle\langle R; \alpha; + \rangle\rangle$ should be sufficiently rich to entail that $s \models (\forall \dot{x} \in U)\langle\langle R; \alpha; - \rangle\rangle$. In contrast, in the general case a situation not supporting a positive valenced infon does not imply that the situation supports its dual, and so the principled argued here for by Devlin is stronger.

Quantifiers modeled as unary relations are situated, and hence range over the objects in a situation. A relation or property, as part of an infon, is factual whenever it is supported by some situation: $s \models \langle\langle \exists^{sit}; T; + \rangle\rangle$ whenever there is an appropriate assignment in s satisfying T , and $s \models \langle\langle \forall^{sit}; T; + \rangle\rangle$ whenever every appropriate assignment in s satisfies T . Since these quantifiers are the relations of infons, these infons may also have a negative polarity: $s \models \langle\langle \exists^{sit}; T; - \rangle\rangle$ whenever there is no appropriate assignment in s satisfying T , and $s \models \langle\langle \forall^{sit}; T; - \rangle\rangle$ whenever every appropriate assignment in s fails to satisfy T ³³.

Non-Wellfounded Infons and Situations

Situation theory is notable in that fairly early in its development it chose to admit non-well-founded sets into the theory in order to accommodate natural descriptions of certain self-referential semantic contents. This means that infons exist that satisfy equations such as the following³⁴:

$$\sigma = \langle\langle \text{Expressible-In-English}; \sigma; + \rangle\rangle.$$

Such infons are sometimes called *hyperinfons*. The relation of being expressible in English is a non-wellfounded relation.

It is also possible to have systems of infons that refer to one another in a circular way. Hyperinfons have been used by situation theorists to tackle paradoxical self-referential phenomena, such as the liar's paradox (Barwise and Etchemendy 1989) and certain well-known paradoxes of common knowledge

³³ Barwise and Cooper (1991, 40) observe that infons with quantification relations and negative polarities do not satisfy the property of persistence, which we will discuss in depth later in the thesis. For example, while $s \models \langle\langle \exists^{sit}; T; - \rangle\rangle$ might be true, in some larger situation s' of which s is part there may be some appropriate assignment f in s' for which $s' \models \langle\langle \exists^{sit}; T; + \rangle\rangle$ and $s' \not\models \langle\langle \exists^{sit}; T; - \rangle\rangle$.

³⁴ This example is borrowed from Seligman and Moss (2011, 264).

(Barwise 1989i). For this reason, situation theory turned away from standard set theory as its modeling paradigm and towards the anti-founded set theory of Aczel (1988). Aczel's set theory replaces the Axiom of Foundation in standard set theory with Aczel's Anti-Foundation Axiom. The result is a well-behaved theory of sets enriching standard set theory with a class of self-referential objects.

The Foundation Axiom and the Anti-Foundation Axiom

The standard axiomatic theory of sets is known as Zermelo–Fraenkel set theory with the axiom of choice, henceforth called ZFC. ZFC is designed to preclude the unrestricted comprehensions leading to Russell's Paradox. One of the axioms of ZFC, but not itself necessary to banishing Russell's Paradox, is called the Axiom of Foundation (or Axiom of Regularity). The Axiom of Foundation says that every set, except the empty set A , has some member disjoint from A . A binary relation R on a set A is wellfounded if there is no infinite sequence a_0, a_1, a_2, \dots of elements of A such that $a_{n+1} R a_n$ for $n = 0, 1, 2, \dots$ (Barwise and Moss 1996 24). An example of a well-founded relation is the order relation on natural numbers. However, the greater-than order on integers is non-wellfounded (Barwise and Moss 1997, 24). Every set A is associated with a structure $\langle A, \in \rangle$ where \in is the membership relation on the elements of A . The Foundation Axiom may be interpreted as asserting that for every set A , the \in relation of the associated structure $\langle A, \in \rangle$ is well-founded (Barwise and Moss 1997, 24).

There are two ways in which a set may fail to be wellfounded (Barwise 1989n, 192): a circular sequence $a \in \dots \in a$ of membership of finite length greater than two, or an ungroundable infinite sequence $\dots a''' \in a'' \in a' \in a$ with no terminating 'bottom-floor' element. Non-wellfounded set theories admit sets that fail to be well-founded. Following the literature, we call non-wellfounded sets

hypersets. Peter Aczel (1988) shows that a non-wellfounded set theory that takes the axioms of ZFC and replaces the Foundation Axiom with his Anti-Foundation Axiom is consistent.

There are several versions of Aczel's Foundation Axiom, and different modes of presentation for each version. This informal presentation of Aczel's Anti-Foundation Axiom with Atoms draws upon or adapts definitions found in Aczel (1988), Barwise (1989n), and Barwise and Moss (1996).

A graph $G = \langle V, E \rangle$ consists of a set V of vertices and set E of directed edges. Instead of writing $E(v, v')$, we will write $v \rightarrow v'$ if there is an edge from v to v' in E . If $x \rightarrow y$ then y is called a child of x . A *pointed graph* is a graph in which one vertex is distinguished, called its *point*. An *accessible pointed graph* (hereafter referred to as APG) is a graph $G = \langle V, E \rangle$ with a distinguished vertex v such that for every vertex $v' \in V$, there is a path from v to v' (Aczel 1988, 4). Following Aczel (1988), we will use APGs as the *pictures* of sets, as defined below.

A *tagged graph* is a graph where each childless vertex v is either tagged with the empty set or an atom. We designate this as $tag(v)$. A *decoration* of a graph G is a function d assigning a set from the universe of hypersets to a vertex of V . The function d must satisfy the condition that:

$$d(x) = \begin{cases} tag(x) & \text{if } \neg \exists y[x \rightarrow y] \\ \{d(y) \mid x \rightarrow y\} & \text{if } \exists y[x \rightarrow y] \end{cases}$$

If $x = \{d(y) \mid y \in V\}$ then G is a *picture* of the set x . Notice that if a graph G is an APG with point p , then G is a picture for the set given by the decoration of p .

Mostowski's Collapsing Lemma states that every well-founded graph has a unique decoration assigning a set to the graph. Given the definition of an APG

above, a corollary to this is that every well-founded APG pictures a unique set (Aczel 1988, 5). Conversely, every set has a picture in some graph³⁵.

Aczel's Anti-Foundation Axiom extends Mostowski's Collapsing Lemma by asserting that that *every graph has a unique decoration*, not just the well-founded ones. Given the Anti-Foundation Axiom, then, every APG pictures a unique set and every non-wellfounded APG pictures a unique non-wellfounded set (Aczel 1988, 6)³⁶.

We give three examples to illustrate how this gives us sets. In our first example, we derive a well-founded set. In the second example we take a look at the Quine Atom. Its non-wellfoundedness arises from circularity. In the third example, we look at non-wellfoundedness arising from an infinite descending chain. In both our second and third examples, the graph pictures the same set Ω .

Example 1.13. Let $\mathbf{G} = \langle G, E \rangle$ be a graph such that $G = \{v, v_1, v_2, v_3, v_4\}$ where v is the point of an APG whose edges are depicted in Figure 1. We let $tag(v_2) = \emptyset$, $tag(v_3) = a$, and $tag(v_4) = b$. Hence the decoration for these three nodes will be the value assigned by their tags, and the decorations for the remaining two nodes are:

$$d(v_1) = \{d(v_3), d(v_4)\}$$

and

³⁵ In fact, there will be an infinite number of graphs picturing any particular set. To see why, note that a vertex in a graph can have an arbitrary number of distinct child vertices tagged with the same value. But, however many child vertices tagged with the same value there might be, no difference in the set assigned by the decoration is the result, for the obvious reason that there is no duplication of elements in a set.

³⁶ This can be strengthened to what Aczel (1988) calls Systems, which are like graphs except that there are a proper class of vertices and a proper class of edges, with the restriction that for any vertex v , the collection of v 's children is a set and not a proper class.

$$d(v) = \{d(v_1), d(v_2)\}.$$

If were to work this out, we would have:

$$d(v) = \{\{a, b\}, \emptyset\}.$$

Clearly the child-of relation is well-founded and the set designated by g is well-founded.



Figure 1. a) Left is a well-founded APG. b) Center is the circular APG picturing Ω . c) Right is the unfolded APG picturing Ω .

Example 1.14. The so-called Quine Atom $x = \{x\}$ defines a non-wellfounded set. The relevant graph of the Quine Atom is depicted in Figure 1b. Its only node is x and its only edge is $x \rightarrow x$. There are no nodes having a tag. Hence the decoration for the node x is $d(x) = \{d(x)\}$. As can be seen, its wellfoundedness fails due to the graph's circularity. Ω is the only set that is its own singleton (Barwise and Moss 1996, 77). An APG pictures Ω iff every vertex in the APG has a child (Aczel 1988, 7). Thus, we can see that there are many APGs that picture Ω . Our next example is one of these.

Example 1.15. Let $\mathbf{G} = \langle G, E \rangle$ be a graph with an infinite indexed set of nodes $G = \{v_0, v_1, v_2, \dots\}_{i \in \mathbb{N}}$ such that $v_0 \rightarrow v_1 \rightarrow v_2 \rightarrow \dots$ as depicted in Figure 1c

where each node v_i is decorated $d(v_i) = \{d(v_{i+1})\}$. Despite the obvious differences between this graph and the graph from our previous example, they both define the set Ω . We may view this APG as an ‘unfolding’ of the previous one (Aczel 1988, 6).

Every APG corresponds to system of equations. This system of equations has a unique solution, the set whose picture is the APG to which the system of equations corresponds.

Identity of Hypersets and Non-Wellfounded Relations

Define \equiv to be the relation between sets such that $a \equiv b$ iff there is an APG that is a picture of both a and b . AFA implies that if $a \equiv b$ then $a = b$. The \equiv relation is a *bisimulation*. A bisimulation R on sets with atoms may be defined as follows:

Definition 1.6 (Adapted from Aczel 1996, 13). Let V_A be a universe of hypersets on a class of urelements or atoms A . A binary relation R on V_A is a *bisimulation* if $R \subseteq R^{+V} \cup R^{+A}$, where for all $a, b \in U \cup A$:

$$aR^{+A}b \Leftrightarrow (a, b \in A \wedge a = b)$$

and

$$aR^{+V}b \Leftrightarrow (\forall x \in a \exists y \in b \ xRy \wedge \forall y \in b \exists x \in a \ xRy).$$

Unification of Parametric Hypersets

The universe of hypersets can be extended to include parameters. Given a set of urelements A called atoms and a proper class of urelements X called parameters, let $V_A[X]$ be the universe of hypersets over $A \cup X$. Sets containing

elements from X we may call parametric sets. Parametric objects can be unified (Barwise 1989a). Define an anchor $f : X \rightarrow V_A[X]$ to be a function from the class of parameters to sets in the universe. Represent a parametric set as $a(\vec{x})$ where \vec{x} designates the collection of parameters occurring in a . An anchor f is a unifier for two parametric objects $a(\vec{x})$ and $b(\vec{x})$ if $a(\vec{x})[f] = b(\vec{x})[f]$. Barwise (1989a, 279) proposes that two parametric sets are unifiable if there is a bisimulation between them. His definition of a bisimulation is slightly different than the one we gave earlier. We give Barwise's definition below:

Definition 1.7 (Barwise 1989a, 279). A *bisimulation* is an equivalence relation \sim on some subclass of $V_A[X] \cup A \cup X$ satisfying the condition that if $a \sim b$ then the following three conditions hold:

1. If $a \in A$ then $a = b$
2. If a and b are sets then $\forall x \in a \exists y \in b \ x \sim y$
3. If \dot{x} is a parameter in the field of \sim , then there is a set c such that $x \sim c$.

Remark. For each of these three conditions, the expected symmetric conditions also follow, since \sim has been defined here as an equivalence relation.

Example 1.16. Let $\dot{x}, \dot{y} \in X$. Let $\{\dot{x}, \{p, \dot{y}\}\}$ and $\{\{\dot{x}\}, \{p, \{\dot{x}, \dot{y}\}\}\}$ be two parametric sets. One way to unify these two parametric sets is by mapping \dot{x} to the hyperset $a = \{a\}$ and \dot{y} to the hyperset $b = \{a, b\}$. Substituting in a and b and unfolding each we get:

$$\{\dot{x}, \{p, \dot{y}\}\}[f]$$

$$\begin{array}{cc}
\{a, \{p, b\}\} & \{\{\dot{x}\}, \{p, \{\dot{x}, \dot{y}\}\}\}[f] \\
\{\{a\}, \{p, \{a, b\}\}\} & \{\{a\}, \{p, \{a, b\}\}\} \\
\{\{\{a\}\}, \{p, \{\{a\}, \{a, b\}\}\}\} & \{\{\{a\}\}, \{p, \{\{a\}, \{a, b\}\}\}\} \\
\{\{\{\{a\}\}\}, \{p, \{\{\{a\}\}, \{\{a\}, \{a, b\}\}\}\}\} & \{\{\{\{a\}\}\}, \{p, \{\{\{a\}\}, \{\{a\}, \{a, b\}\}\}\}\} \\
\vdots & \vdots
\end{array}$$

The notion of a bisimulation is crucial to determining the identity of situation-theoretic objects. For example, two infons are identical if they have the same relation, the same polarity, and if their assignments are identical. However, in the last case, it is important that we be able to determine whether the assignments are in fact the same. For atomic non-structured objects, this is not troublesome. To determine that two structured objects are identical, we must determine that they have the same components built up in the same way. If all such objects are well-founded, this too is relatively straightforward. However, with non-wellfounded objects, a descending chain need not ever terminate at some atom whose identity is plain. Therefore, two infons are said to be the same if they are bisimilar. We will discuss this in more detail in our discussion of infon equality.

Unsaturated Infons

Infons with assignment functions not assigning objects to every argument role of its relation are called unsaturated infons, whereas infons with every role

filled are saturated infons. Some, like Devlin (1991a, 126-127), place constraints on how unsaturated an infon can be before it is no longer well-formed. Devlin calls these the *minimality conditions* of a relation. Unsaturated infons have been found useful for modeling certain kinds of sentences in natural language.

Example 1.17. Consider the relation of forgetting. Since one cannot forget without there being something forgotten, the relation of forgetting will at the very least have roles for both the forgetter and the forgotten. However, while the statement

The password is forgotten.

describes a situation in which a particular password is forgotten, it does not make explicit who has forgotten the password. In this case, it was John who forgot his password. Situation semanticists have chosen to represent the content of such statements through the device of unsaturated infons.

Let us suppose that the relation of forgetting has the following roles: the *forgetter*, the *forgotten*, a *time*, and a *location*. We might therefore have the saturated infon:

$$\phi = \langle\langle \textit{Forget}; \textit{forgetter} \rightsquigarrow \mathbf{John}, \textit{forgotten} \rightsquigarrow \mathbf{password}, \textit{time} \rightsquigarrow \mathbf{t}, \textit{location} \rightsquigarrow \mathbf{I}; + \rangle\rangle$$

This might describe a situation in which John forgot his password at time **t** and location **I**. There are several ways in which the relation of forgetting might be left

unsaturated in an infon. Here are several possibilities. Our first example illustrates how one might indicate the information that the password is forgotten³⁷.

$$\langle\langle \textit{Forget}; \textit{forgetter} \not\rightarrow, \textit{forgotten} \rightsquigarrow \mathbf{password}, \textit{time} \not\rightarrow, \textit{location} \not\rightarrow; + \rangle\rangle.$$

Or a statement might be that

John forgot the password.

but neglect to mention the time or place where it was forgotten, and so we have that the described situation supports the infon:

$$\phi' = \langle\langle \textit{Forget}; \textit{forgetter} \rightsquigarrow \mathbf{John}, \textit{forgotten} \rightsquigarrow \mathbf{password}, \textit{time} \not\rightarrow, \textit{location} \not\rightarrow; + \rangle\rangle.$$

An Order on Well-Founded Infons

The existence of such infons raises several issues about how they may be related and how they may be unified. In fact, an ordering on basic infons may be defined in terms of relative saturation. We may define an ordering on infons by comparing their relations, arguments, and polarities. For example, a plausible and widely adopted (e.g., Seligman and Moss 2011, 279-280) partial ordering on infons may be given roughly as follows: σ is part of σ' , written $\sigma \sqsubseteq \sigma'$, if the relation of σ is the relation of σ' , the polarity of σ is the polarity of σ' , and for each argument a assigned to an argument role i of σ , a is assigned to the same argument role i of σ' ³⁸. Given this ordering on infons, an infon σ is saturated iff

³⁷ From now on, we write *role* $\not\rightarrow$ to indicate that the role is not in the domain of definition of the infon's assignment; if we are leaving the role implicit, we will simply omit mention of the missing assignment and role in the infon.

³⁸ Recall that a relation is associated with a set of argument roles. Therefore two infons with the same relation will have the same argument roles.

$\sigma \sqsubseteq \sigma'$ implies that $\sigma = \sigma'$ for every infon σ' , i.e. there does not exist a σ' such that $\sigma \sqsubseteq \sigma'$ and $\sigma \neq \sigma'$; otherwise it is unsaturated.

It will be noted that such a definition relies upon there being an adequate notion of identity of structured objects. For well-founded structured objects this is straightforward. For non-wellfounded structured objects, we must turn to the notion of a *bisimulation* to determine identity of objects. This requires modifications to our informal definition, but in the interest of clarity, we defer that discussion briefly.

Infon Unification

Two infons are unifiable if there is an upper bound between them in the part-of ordering of infons, and their unification, designated $\sigma_1 \sqcup \sigma_2$ is their least-upper bound³⁹.

Under this definition the infons

$$\langle\langle \text{Eating}; \text{eater} \rightsquigarrow \mathbf{Sheila}, \text{eaten} \rightsquigarrow \mathbf{steak}, \text{time} \not\rightsquigarrow, \text{location} \not\rightsquigarrow; + \rangle\rangle$$

and

$$\langle\langle \text{Eating}; \text{eater} \rightsquigarrow \mathbf{Sheila}, \text{eaten} \not\rightsquigarrow, \text{time} \rightsquigarrow \mathbf{t}, \text{location} \rightsquigarrow \mathbf{l}; + \rangle\rangle$$

are unifiable. Their least upper bound is the infon:

$$\langle\langle \text{Eating}; \text{eater} \rightsquigarrow \mathbf{Sheila}, \text{eaten} \rightsquigarrow \mathbf{steak} \rightsquigarrow \mathbf{t}, \text{location} \rightsquigarrow \mathbf{l}; + \rangle\rangle$$

and so may be unified to form that infon.

³⁹ Hinging upon the part-of relation being appropriately modeled, as we will see.

Logical Import of Unsaturated Infons

Although the relative saturation of infons is a purely structural relation holding between infons, it has ramifications for a consequence relation between infons (Barwise 1989, 180-4). Suppose that σ and σ' are two positively valenced infons such that $\sigma \sqsubseteq \sigma'$. Then σ' obtains with respect to some situation only if σ also obtains with respect to some (possibly different) situation. On the other hand, if the two infons have negative polarities, then the relation goes in the other direction: if σ obtains with respect to some situation then σ' also obtains with respect to some situation.

Given the relationship just described it is reasonable to ask whether or not a situation's support of an unsaturated infon has any quantificational implication. Devlin (1991a, 121) argues that if a situation s supports the positively valenced unsaturated infon $\langle\langle R; \mathbf{a}_1, \dots, \mathbf{a}_m; + \rangle\rangle$ with relation R of arity n , for which the roles $role_{m+1}, \dots, role_n$ are left unfilled (assuming an index for the relation's roles), then there exists some situation s' extending s , and there are objects $\mathbf{a}_{m+1}, \dots, \mathbf{a}_n$ filling the roles $role_{m+1}, \dots, role_n$ such that $s' \models \langle\langle R; \mathbf{a}_1, \dots, \mathbf{a}_m, \mathbf{a}_{m+1}, \dots, \mathbf{a}_n; + \rangle\rangle$, but $s \not\models \langle\langle R; \mathbf{a}_1, \dots, \mathbf{a}_m, \mathbf{a}_{m+1}, \dots, \mathbf{a}_n; + \rangle\rangle$.

In the case of the corresponding negatively valenced unsaturated infon $\langle\langle R; \mathbf{a}_1, \dots, \mathbf{a}_m; - \rangle\rangle$, if $s \models \langle\langle R; \mathbf{a}_1, \dots, \mathbf{a}_m; - \rangle\rangle$ then for every situation s' extending s and for any objects in s' appropriately filling the roles $role_{m+1}, \dots, role_n$, it must be the case that $s' \models \langle\langle R; \mathbf{a}_1, \dots, \mathbf{a}_m, \mathbf{a}_{m+1}, \dots, \mathbf{a}_n; - \rangle\rangle$.

Infon Identity

In situation theory, infons and propositions are structured objects. How are we to judge whether or not two such objects are identical? Earlier we said that two infons σ and σ' are identical if they have the same relation, the same polarity, and the same assignments. We also used this as the basis for building a partial-

order of relative saturation of infons. Unfortunately, when the components of an infon are not well-founded, this simple scheme does not suffice to determine the identity of infons, or any other structured object. We can see this quite clearly in the following example⁴⁰. Let R be a unary relation whose sole role appropriately takes an infon. We will identify that role as the role 1. Consider the following two infons:

$$\sigma = \langle\langle R;1 \rightsquigarrow \sigma;+\rangle\rangle$$

and

$$\sigma' = \langle\langle R;1 \rightsquigarrow \sigma';+\rangle\rangle$$

If we were to pursue the approach described above, in order to determine whether $\sigma = \sigma'$ we must ascertain that the two infons have the same assignments to the role 1. But this clearly begs the question! To answer this question we must ascertain whether σ and σ' are the same infon.

Both infons have the same non-wellfounded structure; in fact they are bisimilar. However, they are distinct infons despite their bisimilarity. However, bisimilarity is as good a concept of identity of non-wellfounded objects as we are likely to get. In order to make it work for us, we must guarantee that our universe does not include distinct bisimilar objects. For this purpose Seligman and Moss (1997; 2011) develop a theory of extensional relations in which to model situation theory.

⁴⁰ This example is adapted from one in Seligman and Moss (2011, 265).

Extensional Relational Structures

A relational structure may be described in various ways. We adopt a description that is a variant of that given in Seligman and Moss (2011, 271-314). A *relational structure* A consists of a universe of objects \mathcal{A} along with a set of relations Rel on \mathcal{A} , and a *relational signature* $\nu: Rel \rightarrow \mathbb{N}$. The number $\nu(R_i)$ is called the *arity* of the relation R_i . Usually we will refer to relations in a structure using an index i and so instead of writing $\nu(R_i)$ we will write $\nu(i)$. The *extension* of a relation R is the various sequences of objects in \mathcal{A} standing in that relation. A relational structure A will be said to have *type* $[\mathcal{A}, Rel, \nu]$ if the universe of A , sometimes written $|\mathcal{A}|$, is \mathcal{A} , the set of relations of A , here written $R(A)$, is Rel , and for each $R_i \in R(A)$ the arity of R_i is $\nu(R_i)$ respectively. We abbreviate this using the convenient notation:

$$A: [\mathcal{A}, Rel, \nu].$$

Usually, we will omit the signature ν from the type, with the understanding that all the relations of A have appropriate arities for the type. Also, instead of writing a name of a set of relations in the type, we will indicate which relations are parts of the type directly.

In their development of a mathematical toolbox for modeling situation-theoretic objects, Seligman and Moss use special relational structures called *extensional relational structures*. Extensional relational structures are relational structures of type $[\mathcal{A}, \mathbf{S}_1, \dots, \mathbf{S}_n; R_1, \dots, R_m]$ where the relations $\mathbf{S}_1, \dots, \mathbf{S}_n$ are distinguished as *structural relations* and where R_1, \dots, R_m are *non-structural relations*. For enhanced clarity, we follow the notational innovation introduced in Ginzburg and Sag (2001). Structural relations will be in bold non-italicized fonts

whereas non-structural relations will be italicized fonts. Also, a semi-colon will be placed between structural and non-structural relations.

The non-structural relations R_1, \dots, R_m are intended to model non-structural properties of the relational structure. For example, place-holders—the parameters hidden inside abstracts in the framework of Ginzburg and Sag (2001)—may be constructed using non-structural relations.

Structural relations are intended to supply the identity conditions for structured (non-atomic) objects of the universe \mathcal{A} . A structural relation \mathbf{S}_i of arity $n+1$ has form $\mathbf{S}_i(x_1, \dots, x_n, a)$ where x_1, \dots, x_n are the *components* of the structured object a . Components of a structured object may be atomic, having no components, or may themselves be structured objects. Seligman and Moss stipulate only that the components of a structured object not be a proper class. Structured objects are uniquely determined by the class of structural relations. Let $\mathbf{StrRel}(A)$ be the collection of structural relations of the relational structure A , and let \vec{x} be any sequence of objects from the universe of A . The relational structure A is extensional if

$$\forall a, b \in \mathcal{A} \left[\left(\forall \mathbf{S} \in \mathbf{StrRel}(A), \forall \vec{x} \in \mathcal{A} \left[\mathbf{S}(\vec{x}, a) \Leftrightarrow \mathbf{S}(\vec{x}, b) \right] \right) \Rightarrow a = b \right],$$

hence dispensing with distinct structurally identical objects.

Seligman and Moss define a projection operation on structural relations that retrieves the class of structured objects at the $n+1$ th coordinate for a given structural relation. Seligman and Moss call these the *structural sorts* of the relational structure A .

Definition 1.8 Structural sorts (adapted from Seligman and Moss 1997, 260; Seligman and Moss 2011, 273). Let S be a structural relation for some

relational structure A . Define \mathbf{S}^* to be the class of objects a such that there exists some sequence of components \vec{x} in the universe of A such that $\mathbf{S}(\vec{x}, a)$ holds. For each structural relation \mathbf{S}_i of A , \mathbf{S}_i^* is a *structural sort* of the relational structure A .

The following is the appropriate definition of bisimilarity for structured objects in a relational structure. In the following definition let $v(i)$ denote the arity of the structural relation \mathbf{S}_i .

Definition 1.9 Bisimulation (adapted from Seligman and Moss 2011, 273).

Let B be a binary relation on the universe of a relational structure

$$A : [\mathcal{A}, \mathbf{S}_1, \dots, \mathbf{S}_n; R_1, \dots, R_m].$$

The binary relation B is called a *bisimulation* on A provided that for all pairs of objects $a, b \in \mathcal{A}$, the following three conditions hold:

1. If $B(a, b)$ and a is atomic then $a = b$,
2. If $B(a, b)$ and a is not atomic then for all structural relations \mathbf{S}_i with $1 \leq i \leq n$, and for all $x_1, \dots, x_{v(i)}$ such that $\mathbf{S}_i(x_1, \dots, x_{v(i)}, a)$ there exist $y_1, \dots, y_{v(i)}$ such that $\mathbf{S}_i(y_1, \dots, y_{v(i)}, b)$ and for $1 \leq j \leq v(i)$, $B(x_j, y_j)$, and
3. If $B(a, b)$ then $B(b, a)$.

For any two objects $a, b \in \mathcal{A}$, a is *bisimilar to b* (in A) if and only if there exists a bisimulation B of A such that $B(a, b)$. A relational structure

$$A : [\mathcal{A}, \mathbf{S}_1, \dots, \mathbf{S}_n; R_1, \dots, R_m]$$

is *extensional* if for every $a, b \in \mathcal{A}$, if a is bisimilar to b then $a = b$.

The main task therefore is to define a set of structural and non-structural relations such that infons are structured by extensional relations. There are several ways that this can be done. One way might be to define a structural relation for the

basic infons, e.g. **BasicInfon**⁴ where **BasicInfon**(r, α, p, σ) where r is a relation, α is an assignment, p is the polarity, and σ is the infon such that **BasicInfon*** is the sort of basic infons. Of course, relations, assignments, and polarities might need structural, or possibly non-structural relations defining them, and appropriateness of assignments must be guaranteed. Alternatively, basic infons can be defined by several structural relations, for each of its parts. Details about how this may be achieved involve more than we have space to present. We urge our readers to consult Seligman and Moss (1997, 2011) and Ginzburg and Sag (2001).

The framework is open enough to give one broad leeway in choosing how to model the objects of situation theory. However, there are numerous and subtle complications arising from one's choices of structural relations used to model the various components of the theory. This can be seen when we reconsider ordering relations on structural object like infons.

Partiality

As we discussed in our section on unsaturated infons, an ordering on unsaturated infons may be given. Given our discussion above, we may have a better way to handle non-wellfoundedness. The following is a general definition that works well in many cases.

Definition 1.10 (Barwise and Moss 2011, 279). Given an extensional relational structure $A : [\mathcal{A}, \mathbf{S}_1, \dots, \mathbf{S}_n; R_1, \dots, R_m]$, $a \sqsubseteq b$ if for $1 \leq i \leq n$ and every sequence of objects \bar{x} of appropriate length, if $\mathbf{S}_i(\bar{x}, a)$ then $\mathbf{S}_i(\bar{x}, b)$.

Remark. The order \sqsubseteq is a partial order.

We can use this ordering relation to redefine our order on infons. To illustrate how this might work, we will define a simple extensional relational structure A with atoms, relations, roles, polarities and infons. However, before we do so, we need a few more definitions.

Definition 1.11 Substitution (based on Seligman and Moss 2011, 283). Let $A : [\mathcal{A}, \mathbf{S}_1, \dots, \mathbf{S}_n; R_1, \dots, R_m]$ be an extensional structure. A *substitution* in A is a partial function $f : \mathcal{A} \rightarrow \mathcal{A}$.

Definition 1.12 f -Simulation (based on Seligman and Moss 2011, 283). Let $A : [\mathcal{A}, \mathbf{S}_1, \dots, \mathbf{S}_n; R_1, \dots, R_m]$ and let f be a substitution in A . A binary relation B extending f on \mathcal{A} is said to be an *f -simulation* if for all $a, b \in \mathcal{A}$ the following two conditions hold:

1. If $B(a, b)$ and $a \in \text{dom}(f)$ then $b = f(a)$.
2. If $B(a, b)$ and $a \notin \text{dom}(f)$ then the three following conditions hold:
 - a) If a is atomic then $a = b$.
 - b) If a is not atomic then for all structural relations \mathbf{S}_i with $1 \leq i \leq n$, and for all $x_1, \dots, x_{v(i)}$ such that $\mathbf{S}_i(x_1, \dots, x_{v(i)}, a)$, there exist $y_1, \dots, y_{v(i)}$ such that $\mathbf{S}_i(y_1, \dots, y_{v(i)}, b)$ and for $1 \leq j \leq v(i)$, $B(x_j, y_j)$ and,
 - c) If a is not atomic then for all structural relations \mathbf{S}_i with $1 \leq i \leq n$, and for all $y_1, \dots, y_{v(i)}$ such that $\mathbf{S}_i(y_1, \dots, y_{v(i)}, b)$, there exist $x_1, \dots, x_{v(i)}$ such that $\mathbf{S}_i(x_1, \dots, x_{v(i)}, a)$ and for $1 \leq j \leq v(i)$, $B(x_j, y_j)$.

When there is an f -simulation B of A such that $B(a, b)$, we say that the object a is f -similar to b in A . We denote by $f.a$ the unique element b that is f -similar to a , for a given f -simulation B , should it exist. A substitution structure is an extensional relational structure for which $f.a$ exists for every substitution f and every $a \in \mathcal{A}$.

Remark. Seligman and Moss (1997, 271-272) point out that many structures of interest to situation theory are substitution structures, but many are not. Set structures, both well-founded and anti-founded, are substitution structures due to set theory's Axiom of Replacement or AFA, but various kinds of information structures may not be substitution structures, especially those for which there are sortal restrictions on substitutions. One example of such a sortal restriction is that an object be appropriate for the role it fills in an infon's relation. Instead, Seligman and Moss argue that all extensional structures are *partial* substitution structures, and suggest that the existence or non-existence of some $f.a$ is best interpreted as the *appropriateness* of the substitution f acting on a (1997, 272).

We are now ready to define a simple extensional infon structure. Let

$$A: [\mathcal{A},_1 \mathbf{Inf}^1, \mathbf{Rel}^2, \mathbf{BasicInf}^4, \mathbf{Pos}^1, \mathbf{Neg}^1; \mathbf{Role}^2, \mathbf{Approp}^2]$$

be such that the following seven conditions hold:

1. $\mathbf{BasicInf}^* \subseteq \mathbf{Inf}^*$, $\mathbf{Pos}^* \cup \mathbf{Neg}^* = \mathbf{BasicInf}^*$, and $\mathbf{Pos}^* \cap \mathbf{Neg}^* = \emptyset$.
2. $\mathbf{Rel}^* \cap \mathbf{Inf}^* = \emptyset$.
3. If $\mathbf{Rel}(\sigma, r)$ then for some a and for some i , $\mathbf{BasicInf}(r, a, i, \sigma)$.
4. If $\mathbf{Rel}(\sigma, r)$ and $\mathbf{Rel}(\sigma, r')$ then $r = r'$.
5. If $\mathbf{BasicInf}(r, a, i, \sigma)$ then $\mathbf{Rel}(\sigma, r)$, $\mathbf{Role}(r, i)$, $\mathbf{Approp}(a, i)$, and $a \in \mathcal{A}$ is the object assigned to i in σ . That it is an assignment is established by:
6. If $\mathbf{BasicInf}(r, a, i, \sigma)$ and $\mathbf{BasicInf}(r, b, i, \sigma)$ then $a = b$.
7. If $\mathbf{BasicInf}(r, a, i, \sigma)$ and if f is a substitution such that $f.a$ exists and $\mathbf{Approp}(f.a, i)$, then there exists an infon $f.\sigma$ such that $\mathbf{BasicInf}(r, f.a, i, f.\sigma)$.

Remark. **BasicInf*** defines the set of basic infons. A basic infon has positive polarity if it is in **Pos*** and a negative polarity if it is in **Neg***. **Inf*** is the class of all infons. Notice that the set of basic infons is a subset of these. In this way, the structure is modular and can be extended to include structural relations for complex infons like conjunctions and disjunctions. **Rel*** is the set of relations, where **Rel**(σ, r) means that the relation **r** is structurally determined by the infons in which it occurs. There is only one relation of an infon. The nonstructural relation *Approp*(a, i) indicates that a is an appropriate assignment to the role i , and *Role*(r, i) indicates that i is a role of the relation r . The last condition is intended to guarantee that our class of infons includes every appropriate variant assignment.

Example 1.18. Consider the two infons $\sigma = \langle\langle R; 1 \rightsquigarrow a; + \rangle\rangle$ and $\sigma' = \langle\langle R; 1 \rightsquigarrow a, 2 \rightsquigarrow b; + \rangle\rangle$. Where a and b are atomic objects. Obviously, the order on relations should have that $\sigma \sqsubseteq \sigma'$. In our framework these infons are structured objects. We will ignore their relations and polarities for the time being, since they do not differentiate these infons, although any full accounting must take into account all relevant structural relations.

We have the following three tuples in **BasicInf**: (1) **BasicInf**($r, a, 1, \sigma$), (2) **BasicInf**($r, a, 1, \sigma'$), and (3) **BasicInf**($r, b, 2, \sigma'$). It is clear that for every sequence of objects in **BasicInf**, if **BasicInf**(\vec{x}, σ) then **BasicInf**(\vec{x}, σ').

Example 1.19. Consider the two infons⁴¹ $\sigma = \langle\langle R; 1 \rightsquigarrow \sigma, 2 \rightsquigarrow b; + \rangle\rangle$ and $\sigma' = \langle\langle R; 1 \rightsquigarrow \sigma; + \rangle\rangle$. We would like to have it that $\sigma' \sqsubseteq \sigma$. Does this fall out as expected? We have the following tuples in **BasicInf**: (1) **BasicInf**($r, \sigma, 1, \sigma'$), (2) **BasicInf**($r, \sigma, 1, \sigma$), and (3) **BasicInf**($r, b, 2, \sigma'$). And so, since these are structural

⁴¹ It is a curious property of these infons that the saturated infon is self-referential, but the unsaturated infon is not self-referential.

relations of an extensional structure, it is clear that we have everything we need to determine the identity of the structured objects, and that in this case for every sequence of objects in **BasicInf**, if **BasicInf**(\vec{x}, σ') then **BasicInf**(\vec{x}, σ). Since there is no difference between these amongst the other structural relations, we know $\sigma' \sqsubseteq \sigma$. Of course, the identity of the infon σ occurring in each infons arguments is determined by their bisimilarity.

In order to more easily define abstraction, substitution, and application internally, Seligman and Moss (1997; 2011, 285) found it more convenient to model basic infons as structurally determined by three structural relations **Ass**² for assignments into an infon, **Rel**² for relations into an infon, and **Inf**¹, where assignments are functions modeled by a function structural relation **Fun**¹ and an function application relation **App**³, which does the work of assigning objects to roles of an infon.

Although Seligman and Moss do not explicitly indicate this, the ordering on infons that we used above is not sufficiently fine-grained to distinguish unsaturated infons from saturated infons in the extensional structure they describe. To see this, note that in their scheme basic infons are structurally determined by **Rel**(r, σ) with r as relation, **Ass**(α, σ) with α as assignment, and **Inf**(σ) (omitting polarity for simplicity). Assignments are functions, and functions are structurally determined, e.g. **Fun**(α) and **App**(a, i, α) for all values a assigned to i by α . But notice that we might have infons σ and σ' , where **Rel**(r, σ), **Rel**(r, σ'), **Ass**(α, σ), **Ass**(α', σ') such that $\alpha \sqsubseteq \alpha'$. We would like that $\sigma \sqsubseteq \sigma'$, but unfortunately this does not fall out from the ordering relation since the ordering relation on assignments is not taken into account. Therefore, we must use some sort of hereditary ordering relation, such as the one described in Seligman and Moss (1997; 2011, 279-280).

The basic idea is that a hereditary order \sqsubseteq^h on an extensional structure is the greatest binary relation (the greatest fixed point) such that if $a \sqsubseteq^h b$ then for every structural relation S in the extensional structure, if $S(\bar{x}, a)$ then there is a sequence of objects \bar{y} such that $S(\bar{y}, a)$ and $x_i \sqsubseteq^h y_i$ for every $x_i \in \bar{x}$ and $y_i \in \bar{y}$. The hereditary order is also a partial order. Given a hereditary order, then the hereditary ordering between assignments will be taken into account in determining the ordering between infons.

The World of Situations and Their Parts

Situation theorists have taken situations to be the principle parts of the world. Jon Barwise warns us, however, that we ought not to cling too closely to our intuitions about spatial and temporal parts (Barwise 1989f, 259; see also Devlin 1991a, 69; Devlin 1991b). The ways in which the world can be divided are much more general. We must therefore consider the nature of situations in more detail.

The Nature of Situations

Although we are told in Devlin (1991a, 11) that a situation may be understood in our everyday sense of the term, Devlin describes situations as mathematical abstractions that cannot be given “a precise definition...in terms of familiar mathematical concepts,” (Devlin 1991b, 32); examples of situations include “simply connected regions of space-time, highly-disconnected space-time regions, contexts of utterance..., collections of background conditions for a constraint, and so on,” (Devlin 1991b, 31). Meanwhile Zalta (1991) constructs an appealing—if somewhat unusual—theory of situations in which situations are non-existent objects encoding the property of satisfying certain collections of infons. In any case, situations are not to be taken as merely descriptions or ways of

talking about things in a world; they *are* parts of a world. If situations are parts of a world, then we may well ask how fine-grained the partitioning of the world can be, if its parts are only situations, or include other sorts of objects, and how the parts of the world relate to one another and to the whole.

On the other hand, if the “world is the totality of facts, not of things” (Wittgenstein 1921), then situations might be thought of consisting of, or determining, collections of items of information (some of which are factual); indeed, many situation theorists have tended toward this view of situations, as we shall see. In this view, our questions about situations as parts of worlds become questions about what infons there are, which collections of infons qualify as situations, and worlds, and which infons are factual.

Situation theorists are divided into two camps. The first camp belongs to the minority who admit non-actual worlds and situations into their models for either philosophical or pragmatic reasons. In this framework, there are many worlds, among which is a distinguished world, the actual world w_a . Besides the actual world, there are the non-actual worlds, of which there are two sorts: the possible worlds and the impossible worlds. The possible worlds are worlds that might have been, and the impossible worlds are the worlds that could never have been. A situation that is part of the actual world is an actual situation. A situation that is not part of the actual world is a non-actual situation. Every non-actual situation is either a possible situation, or an impossible situation. Note, however, that actual situations may be part of both possible and impossible worlds⁴². Also note that any possible world, including the actual world, may be part of some impossible (incoherent) world. An infon is factual in a world w if it is supported

⁴² Barwise’s (1989f) list of branch-points for situation theory includes the choice of whether or not a portion of the world is part of only one world or part of many worlds.

by some situation in w . An infon is factual simpliciter if it is supported by some situation in the actual world w_a .

The second camp consists of those who for philosophical reasons refuse to countenance possible worlds and possible situations, and restrict their models to situations that are part of the actual world. Perhaps a majority of situation theorists may have been in this camp at some time, not surprising since situation semantics was developed as an alternative to the possible-world semantics of the time. Such theorists do not call these situations ‘actual’, since there is no need. In a situation theory where every situation is actual, an infon is factual if it is supported by some situation.

A fundamental notion universally accepted in situation theory is the *Principle of Coherence*.

Definition 1.13 Principle of Coherence (Barwise 1989m, 235). For every pair of dual infons σ and $\bar{\sigma}$, if s is an actual situation, then if $s \models \sigma$ then $s \not\models \bar{\sigma}$.

Remark. If the universe of situations includes possible worlds, then the principle of coherence may be extended to say that given that s is a situation in a possible world, if $s \models \sigma$ then $s \not\models \bar{\sigma}$.

Situation theorists in either camp hold that the actual world and all of its parts are coherent. Possible worlds and all of their parts, for those who accept them, are also coherent. Impossible worlds are incoherent, although many of their parts may be coherent.

The principle of coherence gives us a means of understanding what possible and impossible situations are. A possible situation is a coherent situation, and an impossible situation is an incoherent situation. Actual situations are possible situations that are part of the actual world.

Identity of Situations

In approaching situations as collections of facts, it is useful to define a notion of informational equivalence between situations. Let us say that two situations s and s' are *informationally equivalent*, $s \cong s'$, relative to a scheme of individuation, just in case they support precisely the same information. More precisely, $s \cong s'$ iff for each infon σ , $s \models \sigma$ iff $s' \models \sigma$ (Barwise 1989f, 264)⁴³. Typically, situation theorists have assumed the following *Principle of Extensionality* with regard to the relation of informational equivalence.

Definition 1.14 Principle of Extensionality (Seligman and Moss 2011, 305).

If $s \cong s'$ then $s = s'$.

Remark. This principle is merely the situation-theoretic version of Leibniz's Law, otherwise known as the Principle of the Identity of Indiscernibles.

Being a Part of a World

In situation theory, parts of the world are called situations, and are individuated relative to a scheme of individuation⁴⁴. If there are parts of worlds, then there must be a relation of part-hood. For obvious reasons, situation theorists have generally sought out, if not flat-out assumed, a part-hood relation satisfying

⁴³ Depending upon a number of other choices a theorist might make in modeling situation theory, the collection of infons in the definition of informational equivalence may range over just the basic infons or may also include the complex situations. Parametric infons are not usually included since they are not considered to be properly informational; however unsaturated infons may be included depending on, for example whether a theorist admits them to the theory, or whatever views on unsaturated infons that theorist holds.

⁴⁴ Different schemes of individuation will divide the world differently. A scheme of individuation is in many ways analogous to a perspective; Jeremy Seligman's theory of perspectives is intended to provide a means of bridging the gaps between different ways the world may be classified. Late channel theory can be seen in much the same light.

the properties of a partial order⁴⁵. An element e in a partial order is maximal if for all elements e' , if e is part of e' then $e = e'$. Not every partial order has maximal elements. Indeed, Barwise (1989n) tries to prove that there is no largest situation, that every actual situation is a proper part of the actual world (see also Barwise and Etchemendy 1989; Barwise 1989f), implying that the world is not a situation. But situation theorists have mostly assumed that worlds are the maximal elements of the partial order on the parts of the world, these parts being situations. Note however that, depending on how things are set up, the actual world (or indeed any other possible world) may be a proper part of an impossible world. Thus, it may, again depending on how things are set up, be more correct to say that possible worlds (including the actual world) are maximal *coherent* situations in the partial order; where any larger situation is incoherent. In part, this also hinges on whether one accepts, as most do, the idea that worlds resolve every issue, since if they do not, then there might be coherent worlds larger than, say, the actual world. Barwise (1989f) suggests that the actual world does resolve every issue, but that possible worlds might only resolve a few of them.

If two situations are part of the actual world (locally maximal situation) in the partial order, then they must have an upper-bound in the partial-order. This is codified in the following principle:

Definition 1.15 Principle of Compatibility (Barwise 1989m, 235). For any two actual situations s_1 and s_2 there is an actual situation s such that s_1 is part of s and s_2 is part of s .

⁴⁵ Barwise (1989f) assumes a partial order on situations. In contrast, in (Zalta 1991) a partial order on situations is a provable consequence of his theory of situations as objects encoding internal properties.

Remark. If we admit possible worlds into our theory, then the Principle of Compatibility would state that for any two situations, if s_1 and s_2 are in the same possible world then there is a situation s in that world such that s_1 is part of s and s_2 is part of s .

One obvious question at this point concerns the relationship between the Principle of Extensionality and the part-of relation. To elucidate this relationship, we may define a relation of *infor containment* \sqsubseteq ⁴⁶: $s \sqsubseteq s'$ iff for each infor σ , if $s \models \sigma$ then $s' \models \sigma$ (Barwise 1989f, 265). The \sqsubseteq -relation is clearly reflexive and transitive. If we accept the Principle of Extensionality then \sqsubseteq is anti-symmetric as well, making \sqsubseteq a partial order on situations. The question that must be asked and answered is whether \sqsubseteq correctly models the part-of relation between situations.

Let us suppose that it does. One generally desirable consequence of this is that the supports relation between situations and infons is upwardly persistent.

Definition 1.16 Principle of Persistence (Barwise 1989m, 235). If $s \models \sigma$ and s is part of s' then $s' \models \sigma$.

Intuitively, information is not lost by gaining perspective (Barwise 1989m, 235). The facts on the ground in a walnut orchard in Tulare County do not change simply because the situation under consideration covers all the walnut orchards of California. Infor persistence has been widely seen as a desirable and even as a self-evident property of information.

For this reason, and its apparent simplicity, the \sqsubseteq relation is an appealing choice. Therefore, many situation theorists (e.g., Barwise and Perry 1983; Barwise

⁴⁶ There are actually at least a few variants of this relation, depending on whether one includes or excludes saturated infons and complex infons of various kinds. For example, (Barwise 1989n, 185-186) and (Devlin 1991, 72) appear to specify that the relation covers basic infons, including presumably unsaturated basic infons. On the other hand, (Seligman and Moss 2011) definition of infor containment explicitly allows for complex infons of various sorts, including unsaturated infons.

1989; Barwise and Etchemendy 1989; Devlin 1991a; Seligman and Moss 1997, 2011) have been tempted to adopt the following standard model of situations.

Standard Model of Situations

The most prominent model of situations in the situation theory literature is one in which situations are modeled as sets (or hypersets) of infons from some universe of infons determined by a scheme of individuation. In such a model $s \models \sigma$ iff $\sigma \in s$, although the exact form this takes will depend on the modeling framework used. We will follow Devlin (1991a) in calling these entities *abstract situations* in order to distinguish them from genuine situations (as individuated in some scheme of individuation).

For clarity and concreteness, let us temporarily adopt a convenient framework for modeling situations adapted from Barwise (1989n, 190). We designate Sit to be a collection of situations and Inf be a collection of infons as individuated by some scheme of individuation. For completeness, let us assume that for every infon $\sigma \in Inf$, its dual is also in Inf . Let \sqsubseteq be defined over Sit as before. We will model infons as tuples and situations as sets of tuples. We introduce a unique operation⁴⁷ \mathcal{M} on the objects individuated by some scheme of individuation satisfying the following:

1. If b is neither a situation nor an infon, then $\mathcal{M}b = b$
2. If $\sigma \in Inf$ and $\sigma = \langle\langle R; \alpha; i \rangle\rangle$ then $\mathcal{M}\sigma = \langle R, \beta, i \rangle$, a tuple, where β is a function on the domain of α such that $\beta(x) = \mathcal{M}\alpha(x)$.
3. If $s \in Sit$, $\mathcal{M}s = \{\mathcal{M}\sigma \mid \sigma \in Inf \text{ and } s \models \sigma\}$.

⁴⁷ Uniqueness for well-founded systems is trivial. Aczel's class-fixed-point theorem is required to determine uniqueness for non-wellfounded systems.

It is not difficult to see that $s \sqsubseteq s'$ iff $\mathcal{M}s \subseteq \mathcal{M}s'$, as desired⁴⁸. Within this framework, the corresponding Principle of Persistence in \mathcal{M} asserts the unremarkable fact that if $\mathcal{M}\sigma \in \mathcal{M}s$ and $\mathcal{M}s \subseteq \mathcal{M}s'$ then $\mathcal{M}\sigma \in \mathcal{M}s'$.

Of course, if we were to permit ourselves, we might form arbitrary abstract situations, one for every possible set of infons. Many of these would not correspond to any situation in *Sit*. Nor would all of them be actual, or indeed even possible. For example, we would be free to form incoherent situations that are not part of any possible world, much less the actual world. In essence then, the universe of abstract situations would be given by the class of every subset from the class of infons, and the partial order on situations is merely that defined by the subset relation on sets of infons. This is, in fact, a common way to proceed (e.g., Devlin 1991a, 35; Zalta 1991).

It might seem that \sqsubseteq satisfies the Principle of Compatibility. It does, in fact, satisfy this principle when the class of infons consists only of saturated basic infons. Unfortunately, if the class of infons include unsaturated infons, or some kinds of complex infons, then this principle can only be conserved under the \sqsubseteq -ordering at the expense of persistence or coherence, and if we accept the validity of a perspectival class of infons, then it may be, as some have argued e.g., Seligman and Moss (1997; 2011), that some situations in the actual world are irreducibly incompatible.

⁴⁸ We also might have modeled situations in the extensional relational structure framework of Seligman and Moss (1997; 2011, 305) as follows: Let A be an infon structure, as defined before. We extend A to a *situation structure* by introducing two structural relations **SupportedBy**² and **Sit**¹ such that if **SupportedBy**(σ, s) then **Sit**(s) and **Inf**(σ), and if **Sit**(s) then s is not a relation, function, or infon. Clearly situations are distinguished from each other in such a model precisely by the differences in which infons each situation supports. Clearly also the general part-hood relation \sqsubseteq we defined in this framework is equivalent to the infon-containment relation \sqsubseteq we are describing.

Partiality vs. the Triad of
Compatibility, Coherence, and
Persistence

Unsaturated infons complicate the proper ordering of infons considerably because for any two relatively consistent infons σ and τ there may exist inconsistent infons σ' and τ' such that $\sigma' \sqsubseteq \sigma$ and $\tau' \sqsubseteq \tau$. Generally, the prerequisite for this kind of conflict is that σ' and τ' be identical (up to bisimilarity) in all respects except for their polarity. We illustrate this using several examples. The first example involves changes of state, the second involves relations that depend on a contextual argument, and the third example involves parts of a shared situation.

Example 1.20. Let us suppose that we have some finite state machine \mathbf{m} . Let us suppose that in some actual situation s' the machine is in state 0:

$$s' \models \langle\langle \text{State}; \mathbf{m}, 0; + \rangle\rangle.$$

Let us further suppose that in some other actual situation \mathbf{m} is not in state 0:

$$s'' \models \langle\langle \text{State}; \mathbf{m}, 0; - \rangle\rangle,$$

in this case because

$$s'' \models \langle\langle \text{State}; \mathbf{m}, 1; + \rangle\rangle.$$

Because both s' and s'' are actual situations, then by the principles of compatibility and persistence there must be an actual situation s such that $s' \sqsubseteq s$ and $s'' \sqsubseteq s$, and where:

$$s \models \langle\langle \text{State}; \mathbf{m}, 0; + \rangle\rangle \text{ and } s \models \langle\langle \text{State}; \mathbf{m}, 0; - \rangle\rangle.$$

The situation s is therefore incoherent, and thus an impossible situation in an actual world. This is obviously intolerable; most would agree that it would be far better to either abandon the principles of compatibility or persistence, or to seek out another part-hood relation on situations.

The solution, may have something to do with these being unsaturated infons⁴⁹. For example, if the infons' assignment functions were extended to include the times at which the machine \mathbf{m} was and was not in the states 0 and 1, then it could be the case that we would find that:

$$s \models \langle\langle \text{State}; \mathbf{m}, 0, \mathbf{t}; + \rangle\rangle$$

and

$$s \models \langle\langle \text{State}; \mathbf{m}, 0, \mathbf{t}'; - \rangle\rangle$$

where $\mathbf{t} \neq \mathbf{t}'$.

Example 1.21. Problems of persistence are not restricted to temporal shifts. For example, Devlin (1991a, 126) considers an actual situation in which an individual is alone, and a second situation extending the first such that that individual is not alone.

For example, Amed might be alone in his bedroom, but not alone in his apartment complex. If Amed is alone in his bedroom we have:

⁴⁹ There is a slight embarrassment here that may go unnoticed. On the one hand we defined any particular infon to have a relation of a certain arity and defined an unsaturated infon to be an infon whose assignment function does not assign objects to every argument role of the relation, committing us to the foreknowledge of which infons are saturated and which are not. On the other hand, we are pretending here that we have taken an infon "from the wild", uncertain whether or not it is in fact unsaturated. Is a scheme of individuation beholden to metaphysical plausibility, or to an agent's naivety?

$$s_{bedroom} \models \langle\langle \textit{Alone}; \mathbf{Amed}; + \rangle\rangle$$

and if Amed is not alone in his apartment complex we have:

$$s_{apt. \textit{cmplx}} \models \langle\langle \textit{Alone}; \mathbf{Amed}; - \rangle\rangle.$$

If we accept that $s_{bedroom} \sqsubseteq s_{apt. \textit{cmplx}}$ and if we accept the principle of persistence, then we are led to the inevitable conclusion:

$$s_{apt. \textit{cmplx}} \models \langle\langle \textit{Alone}; \mathbf{Amed}; - \rangle\rangle \wedge \langle\langle \textit{Alone}; \mathbf{Amed}; + \rangle\rangle.$$

Again, the immediate observation that one might make is that these may be unsaturated infons. In particular, one may argue that the relation *Alone* has two roles: (1) that which is alone, and (2) the context in which it is alone. For example,

$$s_{apt. \textit{cmplx}} \models \langle\langle \textit{Alone}; \mathbf{Amed}, \mathbf{bedroom}; + \rangle\rangle \wedge \langle\langle \textit{Alone}; \mathbf{Amed}, \mathbf{apt. \textit{complex}}; - \rangle\rangle$$

is not incoherent⁵⁰.

Example 1.22. Another example of unsaturated infons precipitating a failure of coherence or persistence is the following. Imagine a dinner situation in which Felice and Julia are having dinner together. Felice has the fish, and Julia has the steak. Suppose that we individuate two sub-situations, one having Felice and her meal as its constituents, but not Julia, supporting the unsaturated infon $\langle\langle \textit{Eat}; \textit{eater} \not\rightarrow, \textit{eaten} \rightsquigarrow \mathbf{fish}; + \rangle\rangle$, and the other having Julia and her meal, but not Felice as its constituents, supporting the infon $\langle\langle \textit{Eat}; \textit{eater} \not\rightarrow, \textit{eaten} \rightsquigarrow \mathbf{fish}; - \rangle\rangle$:

$$s_{Felice} \models \langle\langle \textit{Eat}; \textit{eater} \not\rightarrow, \textit{eaten} \rightsquigarrow \mathbf{fish}; + \rangle\rangle$$

⁵⁰ Arguably, this would be better represented using a generalized quantifier. The framework of generalized quantification presented later satisfies persistence, since it is bounded.

and

$$s_{Julia} \models \langle\langle \text{Eat}; \text{eater} \not\rightarrow, \text{eaten} \rightsquigarrow \mathbf{fish}; - \rangle\rangle.$$

It would seem then that the dinner situation of which these two situations are part must support both unsaturated infons, and therefore be incoherent, if persistence and compatibility are maintained.

There is no great mystery here in that no argument is filling the role of *eater* here, and if we fill that role with Felice and Julia in each infon respectively, then the apparent conflict disappears. Unfortunately, that these infons are consistent does not of itself solve the problem of their unsaturated parts.

Situation theorists are well aware of these problems, and have employed a number of strategies to resolve them. Jon Barwise (1989f, 272) attributes to Stanley Peters the suggestion that the principle of persistence be replaced with one which asserts that for all situations and infons if $s \models \sigma$ and a situation s' extends s then there exists a more saturated infon σ' such that $\sigma \sqsubseteq \sigma'$ and $s' \models \sigma'$.

Seligman and Moss (2011, 306-307) take up a version of Peter's proposal. They argue that the infon-containment relation \sqsubseteq between situations as defined in the standard model fails to adequately model the part-hood relation between situations. They propose an additional ordering relation between situations, a part-hood relation, one that explicitly takes into account the ordering relation between infons (such as the one discussed in the sections on saturated and unsaturated infons).

Definition 1.17. A situation s is part of another situation s' , written $s \leq s'$, if for every infon σ , if $s \models \sigma$ then there exists an infon σ' such that $s' \models \sigma'$ and $\sigma \sqsubseteq \sigma'$.

Example 1.23. To illustrate how this works, reconsider the case in which in one situation, Amed is sleeping, and in another, Amed is not sleeping:

$$s \models \langle\langle \text{Sleeping}; \text{sleeper} \rightsquigarrow \mathbf{Amed}, \text{time} \rightsquigarrow +; + \rangle\rangle$$

and

$$s' \models \langle\langle \text{Sleeping}; \text{sleeper} \rightsquigarrow \mathbf{Amed}, \text{time} \rightsquigarrow -; - \rangle\rangle$$

and let s'' be a situation extending s and s' so that $s \leq s''$ and $s' \leq s''$. Instead of s'' supporting these infons, it is only necessary that there be some more saturated infon that is supported, e.g.,

$$s'' \models \langle\langle \text{Sleeping}; \text{sleeper} \rightsquigarrow \mathbf{Amed}, \text{time} \rightsquigarrow \mathbf{t}; - \rangle\rangle$$

and

$$s'' \models \langle\langle \text{Sleeping}; \text{sleeper} \rightsquigarrow \mathbf{Amed}, \text{time} \rightsquigarrow \mathbf{t}'; - \rangle\rangle$$

where $\mathbf{t} \neq \mathbf{t}'$, and so coherence, compatibility, and a form of persistence are maintained.

Properties of the Parthood Relation

Although it is the case that if $s \sqsubseteq s'$ then $s \leq s'$, a result that Seligman and Moss call the *principle of inclusion*, the converse holds only if all the basic infons in s and s' are saturated. Furthermore, under the new \leq relation, the usual notion

of persistence fails to hold unless all infons are saturated (but Stanley Peter's version of persistence does hold, of course).

Unfortunately \leq is not anti-symmetric, and hence not a partial order (Seligman and Moss 2011, 307). The problem is that if one situation s is defined by two infons σ and σ' such that $\sigma' \sqsubseteq \sigma$ and another situation s' is defined by the infon σ , then $s \leq s'$ and $s' \leq s$ but $s \neq s'$. Seligman and Moss propose one way in which a partial order may be preserved: they would require that situations be *relatively saturated*. A situation is relatively saturated if $s \models \sigma$ only if for all $\sigma' \sqsubseteq \sigma$, $s \models \sigma'$ (Seligman and Moss 2011, 307).

Joining Situations

Under the \leq -ordering on situations, a join of two compatible situations must take into account the \sqsubseteq or \sqsubseteq^h ordering on infons. Intuitively the smallest possible join of two compatible situations s_1 and s_2 (which, depending on the actual class of situations one is dealing with, may not actually be available) would be the situation s having all the infons of s_1 and s_2 , except for those unsaturated compatible infons from each, which are replaced by their unification. Without going into tedious detail, we illustrate with an example:

Example 1.24. We will simply model situations as sets of infons in this example. Let $s = \{\langle\langle R; 1 \rightsquigarrow \mathbf{a}, 2 \rightsquigarrow - \rangle\rangle, \sigma_1\}$ and $s' = \{\langle\langle R; 1 \rightsquigarrow, 2 \rightsquigarrow \mathbf{b}; - \rangle\rangle, \sigma_2\}$. Then $s \sqcup s' = \{\langle\langle R; 1 \rightsquigarrow \mathbf{a}, 2 \rightsquigarrow \mathbf{b}; - \rangle\rangle, \sigma_1, \sigma_2\}$. Note, that the join of these two situations does not include the less saturated versions of these infons.

Other Approaches

As Seligman and Moss note, their proposal has not been unanimously accepted. For example, Devlin (1991a, 126-128) argues that every infon's relation

has a subset of its argument roles that must necessarily be filled by some appropriate object (or parameter) if it is to be well-formed, i.e., a genuine infon. Which roles must be filled is determined by the *minimality conditions* of the relation. In particular, those minimality conditions should be sufficient to ensure persistence. Thus, for Devlin infons like $\langle\langle\textit{Sleeping}; \mathbf{Amed}; +\rangle\rangle$, $\langle\langle\textit{Sleeping}; \mathbf{Amed}; -\rangle\rangle$, $\langle\langle\textit{Alone}; \mathbf{Amed}; +\rangle\rangle$, and $\langle\langle\textit{Alone}; \mathbf{Amed}; -\rangle\rangle$ are simply not well-formed⁵¹. Devlin admits, however, that it is not always readily apparent to the theorist whether an infon is in fact malformed. Infons which might seem perfectly acceptable in one context, are not when in some other context. Also, in some cases while positively valenced infons might be persistent, their negatively valenced duals fail to be persistent. However, for Devlin, who is interested in infons being the unit of information, the burden is upon the theorist to ensure that these minimality conditions are met in any model of situation theory.

Quantification, Negation, and Persistence

So far, our discussion has dwelt principally on basic infons. Under the right conditions, persistence of basic infons is guaranteed (at least for saturated infons); however, there are different sorts of difficulties with certain complex infons.

On the face of it, informational phenomena involving terms of generality—i.e., quantifiers, including generalized quantifiers—may fail to be upwardly persistent. Just because there is no x such that it is a cat in my backyard, does not mean that there is no x such that it is a cat in my neighborhood. Just because in one situation every x is such that it is a dog does not mean that in some larger situation every x is a dog.

⁵¹ It is worth emphasizing that one can still have unsaturated well-formed infons, provided that the smallest well-formed infon is not saturated.

Responses to this difficulty have been various. For instance, Barwise (1989m, 234-236) argues that all infons should have negations, including quantified infons. He therefore suggests that situation theorists should come to accept that persistence does not hold of all infons, calling non-persistence no more mysterious than when a property holds of the parts of some whole but fails to hold of the whole itself, as when, he says, borrowing an analogy from John Perry, that a limb may be an arm, but the individual whose limb it is does not have the property that it is an arm (Barwise 1989m, 236). On the other hand, as we have already briefly discussed, Devlin (1991a) seeks to maintain persistence of quantified infons by having the domain of quantification be explicitly restricted to a particular set of individuals.

Another approach is given by Robin Cooper's framework for generalized quantifiers. Cooper's framework of generalized quantification has the advantage of maintaining persistence. Further on in this thesis we will discuss Cooper's framework of generalized quantifiers in more detail.

Miscellaneous Remarks

If we are constructing a class of infons then we know ahead of time just what roles a relation has and just which ones are filled in any particular infon. We can also guarantee that every situation be coherent. We can only then be victims of our own confusion. On the other hand, when we are seeking to represent the information supported and carried by our ordinary discourse and by the world around us, we do not have this advantage. We may be surprised to discover inconsistencies leading to incoherence, but it is not clear that removing these inconsistencies from our representations improves their accuracy. If our guiding notion is a metaphysically plausible or scientifically realistic representation of the

parts of the world, then we may expect there to be no incoherent actual situations, though the universe is certainly a stranger place than our ancestors imagined it to be. But if we take seriously the idea that schemes of individuation are relative to agents—genuine agents, not the avatars of perfect rationality and omniscience that inhabit the worlds of so many theoreticians—then general principles such as coherence, persistence, and compatibility may really have no more weight that the agents happen to give to them, and probably less. There is related tension between metaphysical plausibility and applicability to various sorts of problems (such as in philosophical linguistics).

Perspectival Infons and Situations

We turn now to a highly contested issue, although as we will see, it is closely related to our preceding discussion of unsaturated infons and persistence. We borrow and modify an example from Barwise (1989m, 238-242).

A Short Mystery. Sherlock Holmes, Dr. Watson, and Inspector Lestrade are seated around the table in the drawing room of Holmes' apartment at 221B Baker Street.

“The salt is to the right of the pepper,” said Dr. Watson.

Sherlock Holmes let out a long puff of smoke. “If you observe carefully, Watson, it is also the case that the salt is not to the right of the pepper.”

“Surely not!” exclaimed Watson.

“If you ask me,” muttered Inspector Lestrade, looking baffled, “the salt is in front of the pepper. It's as plain as a pikestaff.”

“It is indeed, my dear Inspector,” said Holmes, graciously.

“By heavens, Holmes, I see what you mean!” shouted Watson, standing up excitedly.

“Sit down Doctor. It is merely a matter of deduction across situated perspectives.”

Perspectival Relations

Infons with perspectival relations have received considerable attention in the literature (e.g., Barwise 1989; Seligman 1990a, 1990b; Seligman and Moss 1997, 2011). Perspectival relations are relations that describe the world from some perspective. These include relations like *LeftOf*, *RightOf*, *Above*, *Below*, *InFrontOf*, *Behind*, *Come*, and *Go*. Infons utilizing these relations may appear to violate the principle of persistence since such infons and their duals may seem to be supported by the same situation.

In our story, from the perspective of Holmes, the salt is to the left of the pepper, from the perspective of Watson, the salt is to the right of the pepper, and from the perspective of Lestrade the salt is in front of the pepper. The three characters have situated perspectives regarding the same pair of objects. From the initial perspective of Watson we have:

$$s_{Watson} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; + \rangle\rangle$$

and also:

$$s_{Watson} \models \langle\langle \textit{LeftOf}; \mathbf{salt, pepper}; - \rangle\rangle \text{ and } s_{Watson} \models \langle\langle \textit{InFrontOf}; \mathbf{salt, pepper}; - \rangle\rangle.$$

From the immediate perspective of Holmes we have

$$s_{Holmes} \models \langle\langle \textit{LeftOf}; \mathbf{salt, pepper}; + \rangle\rangle$$

and also both:

$$s_{Holmes} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; - \rangle\rangle \text{ and } s_{Holmes} \models \langle\langle \textit{InFrontOf}; \mathbf{salt, pepper}; - \rangle\rangle.$$

And finally from the perspective of Lestrade:

$$s_{Lestrade} \models \langle\langle \textit{InFrontOf}; \mathbf{salt, pepper}; + \rangle\rangle$$

and also:

$$s_{Lestrade} \models \langle\langle \textit{LeftOf}; \mathbf{salt, pepper}; - \rangle\rangle \text{ and } s_{Lestrade} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; - \rangle\rangle.$$

Any situation s extending the three situations s_{Watson} , s_{Holmes} , and $s_{Lestrade}$ will need to either support all of these infons, or some version of them in order to maintain basic persistence.

Given our discussion of the failure of basic persistence for unsaturated infons, one might justifiably wonder why perspectival infons should be treated any differently. In particular, one might argue that all perspectival infons properly have argument roles identifying the relevant frame of reference, and that the perspectival infons of our example are all unsaturated infons, and so may be subject to the same treatment as any unsaturated infon. Devlin (1991) would say that these infons have failed to meet the minimality conditions for the relations of *LeftOf*, *RightOf*, *InFrontOf*, etc.

However, arguments can be made that perspectives should receive a distinct treatment in order to account for propositionally distinct but perspectivally similar

attitudes (Barwise 1989m). For example, suppose that, contra to the story we just related, both Holmes and Watson believe that the salt is to the left of the pepper, perhaps because Watson has mistaken the pepper shaker for the salt shaker, and vice versa. The propositional content of Holmes' and Watson's beliefs cannot both be true, yet their beliefs share a similar attitude, namely the belief that the salt is to the left of the pepper. If the content of their beliefs necessarily included a parameter for the perspective, then the common content of their beliefs would not be explicitly apparent.

However, it is worth considering alternate treatments of the problem of perspectives in the situation-theoretic literature, of which there are principally two. The first is found in Barwise (1989m) and further elaborated in Barwise (1989f). The second approach, quite different from that of Barwise, is in Seligman (1990a, 1990b). The latter work proposes a general theory of perspectives and—interestingly—shifts between perspectives, sowing the seeds for a general theory of information flow to be developed jointly by Barwise and Seligman, culminating in Barwise and Seligman (1997). We set this work aside momentarily until our discussion of constraints and the relation of carrying information, since Seligman's approach is in some ways a radical departure from our present discussion.

From Perspectival Relations to
Perspectival Situations

Utterances such as:

The salt is to the left of the pepper.

have frequently been interpreted as having their frames of reference as *unarticulated constituents*. Unarticulated constituents are necessary components of the propositional content of a declarative sentence without which it would have no truth value, but for which there is no morpheme in the sentence whose interpretation would supply that content (Perry 1998a). Frequently in natural language we rely on context in various ways to supply such constituents. For example, if John were to say “It is raining,” one unarticulated constituent without which this statement would not have a truth value is the location where it is raining (Perry and Blackburn 1986). By default, unless the utterance context indicates otherwise, that location will be the same general location of the speaker.

One may be tempted to interpret sentences with unarticulated constituents using unsaturated infons. But there is peril in such a move. In an unsaturated, positively valenced infon, an unfilled role signifies existential quantification, since for that unsaturated infon to be factive there must exist some thing in that role (but not filling that role in the infon), and in an unsaturated, negatively valenced infon, it signifies the negation of an existential quantification, since for a negatively valenced, unsaturated infon, there is no thing in that role (Gawron and Peters 1990a, 19). For example, if the relation is the relation of a giving b to c , then there must be a thing that is given, even if in some infon that argument is left unfilled. Barwise (1989m) is wary of equating unarticulated constituents with unsaturated infons because doing so may attribute to speakers of sentences with unarticulated constituents mental contents that are not theirs. Barwise wishes to keep distinct unarticulated constituents (of which the speaker is cognitively attuned) from background conditions of which they are not aware.

Consider, for example, that modern physics tells us that the relation of simultaneity between two events is actually a three place relation, not a two place

relation. Simultaneity of events is not absolute, but depends upon an inertial reference frame. However, for most speakers the inertial reference is neither relevant nor cognitively salient. Indeed most people are not even aware of the relativity of simultaneity. Hence in their speech acts, the inertial reference frame will not be represented by any morpheme in the utterance. Even if we imagine that in some scientific scheme of individuation, infons with the relation of simultaneity have three argument places, one being the inertial frame of the observer, it still would not follow that the speaker of an utterance regarding the simultaneity of two events need have an inertial frame as part of his or her intentions. Barwise would not call this absence an unarticulated constituent, but would rather locate its absence in an unrecognized constant within the background of a particular situated perspective. Barwise suggests that it is unreasonable to interpret speech acts as if such background conditions were necessarily part of their intentions. It is worth noting that Perry (1998a) disagrees on this small point because he maintains that these are indeed unarticulated constituents, but ones of which a speaker might not be aware. Perry and Blackburn (1986) attempt to clarify this distinction between constituents of which speakers are aware and of those of which speakers are not aware; however, clearly there are differences of opinion not easily resolved.

Barwise's (1989m) alternate analysis is provocative. Barwise wishes to locate what had been deemed unarticulated constituents in the underlying utterance situation. To understand his proposal precisely we will have to develop the technical machinery he brings to bear briefly. Barwise defines a function $Arg : Sit \times Rel \rightarrow pow(Rol)$ mapping each situation and relation to a set of roles. The interpretation of this function is that for each situation and relation, some set of roles must be filled for the infon to be informative in that situation. This should recall the minimality conditions of Devlin (1991a). Devlin's minimality conditions

are intended to guarantee the informativeness or sensibility of infons and to guarantee persistence. However, unlike Devlin, Barwise (1989m) is willing to sacrifice the principle of persistence, and of course, unlike Devlin, Barwise relativizes the conditions to the situations that support them.

A role r is not discriminated in a situation s if there is a constant c such that for every infon in that situation, its assignment function maps the role r to the constant c : $\alpha(r) = c$. With this notion, Barwise uses the *Arg* function to project from a higher-arity infon to a lower-arity infon. If R is a relation and R_{-r} is that relation for which the role r is absent, then the relationship between a situation supporting an infon with the first relation and a situation with the latter is given as (Barwise 1989m, 253):

1. $Arg(s_{-r}, R_{-r}) = Arg(s, R) - \{r\}$
2. $s_{-r} \models \langle\langle R_{-r}; \alpha; i \rangle\rangle$ iff $s \models \langle\langle R; \alpha + (r \rightsquigarrow c); i \rangle\rangle$, where $(r \rightsquigarrow c)$ denotes

that the constant c fills the role r .

Under this definition, it is not necessary that some larger situation support the infon $\langle\langle R_{-r}; \alpha; i \rangle\rangle$, since in a larger situation, the role may have more than one value. Hence, persistence is allowed to fail under this proposal, while maintaining both compatibility and coherence.

We illustrate this by returning to our running example of Holmes, Watson, and Lestrade sitting at the table discussing the relative locations of the salt and pepper shakers. Suppose that for a situation s ,

$$Arg(s, LeftOf) = \{obj_1, obj_2, frame\}.$$

It may be that in s , the role of *frame* has a constant value. Suppose that in this case that its constant value corresponds to the perspective of Holmes. Then we may project the relation *LeftOf* to a relation *LeftOf-Frame* and situation s to a situation

s_{-Frame} such that $Arg(s_{-frame}, LeftOf_{-frame}) = \{obj1, obj2\}$,
 $s_{-frame} \models \langle\langle LeftOf_{-frame}; \dot{x}, \dot{y}; + \rangle\rangle[f]$ iff $s \models \langle\langle LeftOf; \dot{x}, \dot{y}, \mathbf{Holmes}; + \rangle\rangle[f]$ for each
 appropriate anchor f , and in particular: $s_{-frame} \models \langle\langle LeftOf_{-frame}; \mathbf{salt, pepper}; + \rangle\rangle$ iff
 $s \models \langle\langle LeftOf; \mathbf{salt, pepper, Holmes}; + \rangle\rangle$.

What then are the differences between this proposal and the previous one? The principle difference is that in the present proposal, the roles of a situation are determined by the situation. One small difference is that the infon $\langle\langle R_{-r}; \alpha; i \rangle\rangle$ is not unsaturated, since the role r is discarded as it has an indiscriminable constant value filling it. This allows us to represent an agent's perspective without suggesting that the agent is in any way cognizant of the omission. It is also assumed that if a situation s is part of a situation s' , then $Arg(s, R) \subseteq Arg(s', R)$, and that if $Arg(s, R) = Arg(s', R)$ for all situations s' of which s is part, then R is non-perspectival in s (Barwise 1989f, 268-9).

Note also that under this proposal, the definition of saturation itself is modified. Namely, an infon with relation R in a situation s is saturated if $Arg(s, R) \subseteq dom(\alpha)$, where α is the assignment function for the infon, and is unsaturated otherwise (Barwise 1989f, 272). In so doing, Barwise (1989f, 272) suggests that his introduction of unsaturated infons into situation semantics had been a mistake that has caused situation theorists endless trouble, and wonders whether a situation can *support* unsaturated infons at all (or at least certain kinds of unsaturated infon). In this, his proposal is a substantive departure from the mainstream of situation theory. Barwise (1989f, 272-6) reminds us that he introduced unsaturated infons in order to model information such as the fact that George is not reading anything at all, e.g.:

$\langle\langle Reading; \mathbf{George}; - \rangle\rangle$

without having to explicitly indicate everything that George is not reading.

Barwise (1989f) argues that instead of a situation supporting an unsaturated infon such as the one above, the possibility of which he had begun to doubt, a situation's being some way might instead *carry* information of a certain sort. In this case, a situation being such that George was doing anything but reading would carry the information that $\sigma = \forall \dot{x} \langle \langle \textit{Reading}; \mathbf{George}, \dot{x}; - \rangle \rangle$ by a negative constraint.

Nonetheless, it is not overwhelmingly clear whether these differences are differences that make much a difference; nor is it clear how much is gained. In the end, Barwise (1989f, 264-5, 269) briefly entertains a third approach utilizing agent-relative constituent functions to populate infons and situations with objects. He does not elaborate much further, except to note that “the route of allowing multiple constituent functions may be a more productive one, but I am not convinced enough to rule the perspectival relations out of court,” (Barwise 1989f, 269).

In this work Barwise does not address how to understand the sorts of shifts *between* perspectives we find in our story of Holmes, Watson, and Lestrade. It is only because they are able to ‘see’ their common situation ‘through another’s eyes’ that they are able to arrive at mutual understanding. How would this be achieved, if it were truly the case that the frame of reference subtracted out from an agent’s perspective was an indiscriminable constant? Furthermore, factive shifts in perspective involve more than mere substitutions of one reference frame value for another in an infon. The proper place for a theory of shifts in perspective must involve some sort of constraint *between* situations *within* some context type. This is precisely what Jeremy Seligman’s theory of perspectives does. We will however defer our discussion of Seligman’s work until after our discussion of constraints and information flow.

SITUATION SEMANTICS

Brief Introduction to Situation Semantics

Our thesis does not have at its focus the myriad concerns of situation semanticists. Rather our attention has been on the formal apparatus developed by situation theorists. Nonetheless, the analysis of natural language has always been the principal application for which situation theory was designed, and inevitably many of the questions, concerns, and ideas coming out of the analysis of natural language have found their way into the theory¹. Therefore in this section we will introduce a few of the main idea of situation semantics, and well as introduce several specific topics within situation semantics that we have found to be of interest. We do not pretend that this introduction is as complete as the topic merits; indeed, we believe that the writing of a survey of situation semantics is long overdue.

Situation semantics is based on two basic ideas: that the meaning of sentences is relational and that information states are partial. We have already described what this partiality amounts to in the theory. We will now describe how situation semantics is a relational theory of meaning; this description will motivate our subsequent discussion of information flow and constraints.

Meaning

What is the meaning of an assertive sentence taken to be in situation semantics? Roughly, it is a relation between a discourse or utterance situation and

¹ At the very least, it might be said that situation semanticists, through their innovations, and for good reasons of their own, created many of the messy problems (e.g. unsaturated infons) situation theorists interested in strong mathematical foundations had to fix up. Note: many of the former were also the latter.

the situation that the sentence describes (Barwise and Perry 1983; Devlin 1991, 87-90):

$$u[[\Phi]]d$$

where Φ is a sentence, and u and d are the relata standing for the utterance and described situations.

The meanings of natural language sentences depend on the contexts of their utterance. An obvious example is the pronoun “I”, which could have as its denotation many different individuals, depending, conventionally, on just who utters that pronoun. Sentences like

He is the winner!

will have many possible interpretations, not least which will depend on just who HE is or at what he is the winner. Often the context-dependency of meaning is even more radical. In situation semantics, context is not just one situation, but a “constellation of situations,” (Gawron and Peters 1990a, 27). The determination of what constellation it is in any particular case depends on a number of factors, including the sentence itself.

In general there will be a *discourse situation* (or *utterance situation*). A discourse situation is the situation in which the utterance is embedded. Among other things, the discourse situation will support various properties of the utterance. In addition to the discourse situation, the context relevant to the utterance of a sentence will frequently include various *resource situations* (of which there are various sorts); resource situations are generally disjoint from the discourse situation. For example, the meaning of the sentence

The woman who authored *The Left Hand of Darkness* is at the airport.

depends upon a resource situation picking out the particular woman being indicated, i.e., the resource situation supporting the fact that the woman authored *The Left Hand of Darkness*.

Several different contents can be associated with an assertive utterance like the one above (Barwise 1993, 4). The *propositional content* of an assertive statement like the one above is a proposition concerning the described situation: in this case, the described situation is described as supporting the information that Ursula LeGuin is at the airport. The *demonstrative content* of the statement is the described situation. The *descriptive content* of the statement consists of the types (or infons) used to describe the described situation.

Perhaps the best way to show how this works is through an example.

Example 2.1. The meaning of the sentence

$\Phi =$ Miriam did not argue.

can be understood as the relation $\llbracket \Phi \rrbracket$ between utterance situations and described situations satisfying:

$$\begin{aligned}
 u \llbracket \Phi \rrbracket d \text{ iff} \\
 & u \models \langle\langle \text{RefersTo}; \text{"Miriam"}, \dot{x}, \mathbf{t}_u; + \rangle\rangle \wedge \\
 & u \models \langle\langle \text{IsUttered}; \Phi, \mathbf{t}_u; + \rangle\rangle \wedge \\
 & u \models \langle\langle \text{CurrentTime}; \mathbf{t}_u; + \rangle\rangle \wedge \\
 & u \models \langle\langle \text{RefersTo}; \text{PastTenseOfVerb}, \dot{i}; + \rangle\rangle \wedge \\
 & d \models \langle\langle \text{Argue}; \dot{x}_{\langle\langle \text{Named}; \dot{x}, \text{"Miriam"}; + \rangle\rangle}, \dot{i}_{\langle\langle \text{Precedes}; \dot{i}, \mathbf{t}_u; + \rangle\rangle}; - \rangle\rangle
 \end{aligned}$$

for some anchor f . Note the use of restricted parameters in the infon supported by the described situation. These are supported by separate resource situations r and r' , relative to the anchor f .

$$r \models \langle\langle \text{Named}; \dot{x}, " \text{Miriam}"; + \rangle\rangle[f] \text{ and } r' \models \langle\langle \text{Precedes}; \dot{t}, \mathbf{t}_u; + \rangle\rangle[f]$$

In this example, d is the demonstrative content of Φ , the infon

$\langle\langle \text{Argue}; \dot{x}_{\langle\langle \text{Named}; \dot{x}, " \text{Miriam}"; + \rangle\rangle}, \dot{t}_{\langle\langle \text{Precedes}; \dot{t}, \mathbf{t}_u; + \rangle\rangle}; - \rangle\rangle[f]$ is the descriptive content of Φ , and the proposition $d \models \langle\langle \text{Argue}; \dot{x}_{\langle\langle \text{Named}; \dot{x}, " \text{Miriam}"; + \rangle\rangle}, \dot{t}_{\langle\langle \text{Precedes}; \dot{t}, \mathbf{t}_u; + \rangle\rangle}; - \rangle\rangle$ is the propositional content of Φ .

The meaning of a sentence then is the result of there being natural and conventional constraints between types of situations. Information flows from utterance situations of one type to described situations of another type. A primary task of situation theory then has been to find an adequate account of information flow by constraints.

Basically a constraint is a relation holding between types of situations such that if a situation is of one type then some other related situation of the other type. Suppose that $T \Rightarrow T'$ is a constraint, where T and T' are situation types. Then if a situation s is of type T , then there is some situation of type T' . We will describe constraints in much detail later.

Meaning of Conditionals

Our discussion is necessarily brief. Nonetheless, it is requisite that we give some indication of a situation-semantic interpretation of conditional statements. The analysis of conditional statements has been a near constant pre-occupation of a great many philosophers, logicians and artificial intelligence researchers for many years. Although the material conditional of classical logic captures much of

what conditional statements mean in natural reasoning, in many respects it falls short. For example, a classical analysis treats any conditional statement whose consequent is necessarily true as a theorem, such as:

If Joe likes pepperoni on his pizza then pi is an irrational number.

Since pi is an irrational number under any circumstance (or any possible world), then this conditional is tautologous; likewise, if the antecedent of a conditional is necessarily false, then the conditional is true. These and other problems like it are often called the *paradoxes of material implication*.

In situation semantics, a conditional statement has as its descriptive content a constraint between types of situations:

$$T \Rightarrow T'$$

(Barwise 1989b). Its demonstrative content would be whatever object d it is that makes that constraint factual, and its propositional content is that for object d , a particular constraint holds (Devlin 1991a):

$$d \models (T \Rightarrow T').$$

Example 2.2. Suppose that we have the conditional statement:

If George sniffed the pepper then he sneezed.

where²

$$[\dot{s} \mid \dot{s} \models \langle\langle \textit{Sniffed}; \mathbf{George, pepper}; + \rangle\rangle]$$

is the type of situation in which George sniffed pepper and

$$[\dot{s} \mid \dot{s} \models \langle\langle \textit{Sneeze}; \mathbf{George}; + \rangle\rangle].$$

is the type of situation in which George sneezed. Then the descriptive content of the statement is that there is the constraint:

$$[\dot{s} \mid \dot{s} \models \langle\langle \textit{Sniffed}; \mathbf{George, pepper}; + \rangle\rangle] \Rightarrow [\dot{s} \mid \dot{s} \models \langle\langle \textit{Sneeze}; \mathbf{George}; + \rangle\rangle]$$

Its demonstrative content is whatever described entity d it is that makes the constraint factual, and the propositional content is that d supports that constraint. We are being purposefully vague on just what sort of entity it is that d would be, because there is no agreement on whether they are situations, the world, or other sorts of entities. What they are will depend upon the exact theory of constraints that is adopted.

This understanding of conditional statements begins to step away from the paradoxes of the material conditional. Conditional statements are true just in case the constraints they describe are factual; constraints are facts about the world, not logical relationships between propositions.

² For simplicity, we have omitted any arguments for time and place, although any serious modeling of these events would require them with additional constraints placed on the relative order of each event.

Generalized Quantifiers in Situation Theory

Situation semantics requires a more expressive set of quantifiers than any we have yet described. A great variety of terms of generality exist in natural language, e.g., *most of*, *few of*, *at least three*. Many of these are not adequately modeled by standard quantifiers or sentences in standard predicate calculus.

The modern study of generalized quantifiers in predicate logic begins with Mostowski (1957), but it is widely recognized (e.g. Glanzberg 2006) that the study of generalized quantifiers in natural language begins with Barwise and Cooper (1981), Higginbotham and May (1981), and Keenan and Stavi (1986). Useful introductions to the literature on generalized quantifiers include Keenan and Westerståhl (2011), Westerståhl (2011), and Glanzberg (2006)³.

We now give a brief informal introduction to generalized quantifiers. We begin with generalizations of the standard quantifiers in first order logic, then discuss the relational model of quantifiers in natural language initially developed by Barwise and Cooper (1981), and finally, we outline how generalized quantifiers might look like within the situation-theoretic framework.

Mostowski Quantifiers

In first order logic, the extension of a unary predicate is a subset of a domain of discourse \mathcal{M} . For example, in a model \mathbf{M} the predicate $P(x)$ denotes the set of elements a from the universe of discourse \mathcal{M} for which $P(a)$ is true. The quantifiers \forall and \exists are understood as properties of such sets. For example, a model satisfies the formula $\forall xP(x)$ if for every variable assignment assigning some a from the universe of discourse to x , $P(a)$ is true. But then the extension of

³ It is remarkable, but not altogether surprising, that many of the most prominent situation semanticists and situation theorists, e.g., Jon Barwise, Robin Cooper, Lawrence Moss, and Dag Westerståhl are also so prominent within the literature on generalized quantifiers in natural language.

$P(x)$ is the universe of discourse. Extensionally, then, the semantic content of the universal quantifier is the set of sets identical to the universe of discourse \mathcal{M} . On the other hand, a model satisfies the formula $\exists xP(x)$ if in that model the extension of $P(x)$ is non-empty. Extensionally, then, the semantic content of the existential quantifier is the set of non-empty subsets of the universe of discourse. More generally, any (local) quantifier is a set of subsets of the universe of discourse, and therefore the universal and existential quantifiers are merely special cases of a more general notion, that of the generalized quantifiers. For example, we could define a quantifier $(Q_R)_{\mathcal{M}}$, known as the Rescher quantifier, as (Glanzberg 2006):

$$(Q_R)_{\mathcal{M}} = \{X \subseteq \mathcal{M} \mid |X| > |\mathcal{M} - X|\}$$

Quantifiers such as these are usually either called *generalized quantifiers* or *Mostowski quantifiers* in recognition of the foundational work in Mostowski (1957), and are said to be of type $\langle 1 \rangle$, taking one set as its input⁴. Mostowski quantifiers may be used to enrich first-order logic and other logical languages.

As it turns out, the proper representation of quantifiers in natural language requires more sophisticated machinery. In particular, quantification in natural languages is a binary relation between two predicates, and thus extensionally, between two sets in some universe. For example, consider these sentences:

Every man is a mortal.

Most dogs chase cats.

⁴ The 1 in the angle-brackets denotes that it takes a set.

Between five and thirty animals escaped the zoo.

We may easily see the binary structure by the following expansions of these sentences:

Every man is a man who is mortal.

Most dogs are dogs that chase cats.

Between five and thirty animals are animals that escaped the zoo.

A sentence in natural language may be parsed as consisting of a noun phrase and a verb phrase. For example, in the second example, the noun phrase is MOST DOGS and the verb phrase is ARE DOGS THAT CHASE CATS. The noun phrase can be further subdivided into a *determiner* and a *common noun*. In this case the determiner is the word MOST and the common noun is DOGS. Roughly speaking, the determiner is the quantifier⁵, and the quantifier ranges over the common noun.

Quantifiers such as these are not modeled best by sets of sets, but by relations between two subsets of the universe of discourse. Quantifiers of this type are said to be of type $\langle 1,1 \rangle$ since they take two sets.

For example, we might model the notion of *most of A are B* as follows (Glanzberg 2006):

⁵ This is actually somewhat controversial. An entire noun phrase may be regarded as a type $\langle 1 \rangle$ quantifier, and Barwise and Cooper (1981) insist on only qualifying noun phrases as quantifiers.

$$Most_{\mathcal{M}}(A, B) \text{ iff } |A \cap B| > |A - B|$$

It is possible to construct many type $\langle 1,1 \rangle$ quantifiers from type $\langle 1 \rangle$ quantifiers. For example, we can express the idea that every man is mortal by the first-order formula: $\forall x[man(x) \rightarrow mortal(x)]$. However, it turns out that not all type- $\langle 1,1 \rangle$ quantifiers can be constructed from type- $\langle 1 \rangle$ quantifiers. In fact, the type- $\langle 1,1 \rangle$ quantifier $Most_{\mathcal{M}}$ that we just gave cannot be so constructed (Glanzberg 2006).

Situation-Theoretic Generalized Quantifiers

Significant contributions on generalized quantifiers within situation semantics include the work of Barwise and Perry (1983), Cooper (1987, 1991, 1993, 1995), Devlin (1991a), and Gawron and Peters (1990a; 1990b). Situation semanticists have presented a number of competing models of generalized quantifiers within the framework of situation semantics. It is beyond the scope of this thesis to explore these in depth. However, these approaches have much in common.

We indicate the general flavor of generalized quantifiers within situation semantics using the approach adopted in Cooper (1995). Cooper's approach is based on the approach of Gawron and Peters (1990a). Under this approach, a generalized quantifier relation is represented as a binary relation Q between (object) types and properties. Situations are involved in a generalized quantifier in three places (Cooper 1995, 5): a described situation d supporting the quantified infon, which Cooper calls the *quantificational situation*, a resource situation r that is part of the object type, and implicit situations i called *individual situations* in which the property of the quantifier relation holds.

Let τ be an object type and let ρ be a property. For each quantifier relation Q , there is an associated set-theoretic quantifier relation Q^* satisfying the following conditions:

$$\exists d[d \models \langle\langle Q; \tau, \rho; + \rangle\rangle] \text{ iff } Q^*(\{a \mid a: \tau\}, \{a \mid \exists i[i \models \rho.a]\})$$

and

$$\exists d[d \models \neg\langle\langle Q; \tau, \rho; + \rangle\rangle] \text{ iff } \neg Q^*(\{a \mid a: \tau\}, \{a \mid \exists i[i \models \rho.a]\})$$

where d is the quantification situation, i is the individual situation, and $\rho.a$ is the infon resulting from application of the property ρ to a , replacing the abstracted parameter of the property with a (Cooper 1995, 4-5). The object type τ will have some situation r as the supporting situation of the object type (not shown).

Example 2.3. We illustrate this with the following example⁶.

Most of the dogs are running.

In keeping with the idea that this sentence corresponds to:

Most of the dogs are dogs that are running.

we construct our quantifier relation as follows:

⁶ As advocated in Cooper (1991a, 299-300), we use the present perfect rather than the simple perfect here to emphasize that the statement describes a situation where most dogs are running, rather than some constraint holding of dogs generally.

$$\exists d [d \models \langle\langle \text{Most}; ([\dot{x} | r \models \langle\langle \text{IsDog}; \dot{x}; + \rangle\rangle], [\dot{x} | \langle\langle \text{Running}; \dot{x}; + \rangle\rangle]); + \rangle\rangle]$$

if and only if

$$\text{Most}^* (\{\dot{x} | r \models \langle\langle \text{IsDog}; \dot{x}; + \rangle\rangle\}, \{\dot{x} | \exists i [i \models \langle\langle \text{Running}; \dot{x}; + \rangle\rangle]\})$$

if and only if

$$|A \cap B| > |A - B|$$

where $A = \{\dot{x} | r \models \langle\langle \text{IsDog}; \dot{x}; + \rangle\rangle\}$ and $B = \{\dot{x} | \exists i [i \models \langle\langle \text{Running}; \dot{x}; + \rangle\rangle]\}$.

The distinction between the described situation (quantificational situation) and the resource situation affords situation-theoretic generalized quantifiers an extra degree of freedom permitting situation semantics to model a number of generalized quantifiers in natural language correctly ⁷. Furthermore, it allows situation theory to retain persistence for quantified infons because the scope of a quantifier is not determined by the situation the infon supports, but by the fixed resource situation (Cooper 1995, 10). Cooper also discusses quantification over resource situations.

⁷ For example, in analyzing clauses such as EVERYTHING IS IN THE BAG or THE MAN SHOOK HANDS WITH THE MAN, if the described situation and the resource situation were the same, then it is difficult to avoid interpreting these sentences respectively to mean that everything (including the bag) is in the bag, and that the man shook his own hand. In the latter case, the uniqueness required by the definite article THE means that the two men cannot both be in the same resource situation. For more information on examples like these consult Cooper (1995) or any of the other major references already given on generalized quantifiers in situation semantics. This use of distinct resource situations for definite descriptions goes some way to answer the criticisms of Soames (1985; 1986). See Barwise and Perry (1983), Kratzer (2009) for further discussion.

Further Reading in Situation Semantics

This is far too brief an explanation of situation semantics. For those interested in further explanation of situation semantics we recommend Barwise and Perry (1983), Barwise (1989), Cavedon (1995), Devlin (1991a), Devlin (2006), Gawron and Peters (1990a), Ginzburg and Sag (2001), and Kratzer (2009).

For our purposes here, it is enough to point out that situation semantics' analysis of the meaning of sentences as relations between utterance situations and described situations is a special case of a more general theory of information flow. However, working out an adequate model of information flow has been no easy task. Most of the remainder of our thesis will be to describe the various attempts at understanding how meaning and information flow arise from constraints. We turn to that discussion now.

INFORMATION FLOW

Introduction to Information Flow

From our discussion so far, the reader might be forgiven for not thinking that the concept of information flow is not at the heart of the situation-theoretic (and the situation-semantic) project. In situation theory, information flow has a particular meaning: information flow occurs when information about one situation is informative about some other situation¹. The information supported by a situation describe that situation, but the world is such that the information supported by a situation can also indicate to us something about other, possibly disjoint or even remote, situations. We therefore must distinguish between the information supported by a situation, and information *carried* by that situation, which we call the flow of information. The question is, how? Like most aspects of situation theory, models of information flow have been various and rapidly evolving.

Reliability of Information Flow

Information, it is argued, is veridical; misinformation is not a species of information, because it does not inform; it misinforms (Dretske 1981). Put in probabilistic terms, if a being F carries the information that b is G , then the probability of b 's being G given that a is F must be 1, for if it were anything less than 1 it would not be knowledge. In order to guarantee this rigorous condition, Dretske proposes that information flow is transitive.

¹ Information flow, as it is understood here, should not be mistaken for the dissemination of information-bearing vehicles (i.e., signals) such as email; it is more general than that.

Definition 3.1 Dretske's Xerox Principle (adapted from Dretske 1981). If a 's being F carries the information that b is G , and b 's being G carries the information that c is H , then a 's being F carries the information that c is H .

Remark. Within the domain of situation theory we may interpret Dretske's Xerox Principle by taking a , b , and c to be situations, and F , G , and H to be situation types.

There are several criteria by which we may judge the adequacy of a theory of information flow. These include whether or not it can guarantee the flow of information in the sense that Dretske argues is necessary, how specific the resulting information flow is, and how gracefully and coherently exceptions and other forms of misinformation are modeled. Many situation theorists argue that any theory of information flow should satisfy some variant of the Xerox Principle (e.g. Dretske 1981; Barwise and Perry 1983; Israel and Perry 1990). The formulations of the Xerox Principle in situation theory are typically more fine-grained than the above. The situation-theoretic account of information flow by simple involvement decomposes Dretske's principle into several parts.

Classic View of Information Flow in Situation Theory

According to situation theory's classic view of information flow, a situation carries information about other situations in virtue of *constraints* (relations of involvement) holding between various situation types. A simple constraint $T \Rightarrow T'$ is a factual relation (or true relational proposition)² between two situation types T , and T' . If $T \Rightarrow T'$ then the situation type T is said to *involve* the situation type T' modulo a common anchor to any of the parameters of the

² Sometimes the information that one type involves another is indicated by the infon $\langle\langle \text{Involves}; T, T'; + \rangle\rangle$, presumably supported by the world.

constraint. The relation of involvement has an existential significance, called the *principle of involvement*:

Definition 3.2 Principle of Simple Involvement (Barwise and Perry 1983; Devlin 1991a, 94; Seligman and Moss 2011, 316). If $s : T[f]$ for some appropriate anchor f and $T \Rightarrow T'$ then there is a compatible situation s' (in the same world) such that $s' : T'[f]$.

Remark. The anchors are necessary because situation types may be parametric abstracts.

The principle of involvement says that if a constraint holds, then if a situation s is of type T relative to an anchor f then there is some compatible situation s' of type T' (relative to f) in the same world (possible or actual). Note that the situation s may not be distinct from the situation s' , and that the precise identification of the situation s' may not be available. If we assume the persistence of infons then the principle of simple involvement may be restated to mean that if $s : T[f]$ for some appropriate anchor f , and $T \Rightarrow T'$ then $w : T'[f]$, where w is a maximal world and where $s \leq w$.

In this context, Dretske's Xerox principle may be understood to mean that the relation of involvement is transitive:

Definition 3.3 Xerox Principle with Simple Involvement (Barwise and Perry 1983, 111; Seligman and Moss 2011, 318). Xerox If $T \Rightarrow T'$, $T' \Rightarrow T''$ then $T \Rightarrow T''$.

Together these two principles imply what we might call the *extended principle of simple involvement*:

Proposition 3.1 Extended Principle of Simple Involvement. If $T \Rightarrow T'$, $T' \Rightarrow T''$, and for some anchor f , $s : T[f]$ then there is a situation s'' (compatible with s) such that $s'' : T''[f]$.

For a constraint $T \Rightarrow T'$, one can discern two contents that can flow. The first of these is the item of information given by $Cond(T')[f]$, which is the non-parametric conditioning infon of the type. Thus, for example Barwise (1989f, 274) and Seligman and Moss (2011, 318-319) indicate that a situation s carries the information $Cond(T')[f]$. The second kind of content is propositional content. For a constraint $T \Rightarrow T'$, the propositional content carried by $s \models Cond(T)[f]$ is the proposition that there is a s' such that $s' \models Cond(T')[f]$ (or that $s' : T'[f]$). Israel and Perry (1990) call this propositional content *pure information*, which may be defined as follows:

Definition 3.4 Pure Information (Israel and Perry 1990, 9-10). Given a simple constraint $T \Rightarrow T'$, for every anchor f , an infon $Cond(T)[f]$, supported by a situation s carries the *pure information that* $\exists s'(s' \models \exists a_1 \dots a_n (Cond(T')[f]))$.

Remark. Israel and Perry include an existence clause for the objects $a_1 \dots a_n$ assigned by f to the roles of the parameter-free infon $Cond(T')[f]$.

An agent *attuned* to a constraint $T \Rightarrow T'$ can infer from information that s is a situation of type T (relative to some f) that there is some situation s' of type T' (relative to some f). Information flows to which an agent is not attuned are not accessible to that agent. Information, as John Barwise once wrote, “travels at the speed of logic, genuine knowledge only travels at the speed of cognition and inference,” (Barwise 1989i, 204).

Example 3.1. To make these notions concrete, suppose that T is a type of situation in which smoke is present, and that T' is the type of situation in which fire is present³. We may suppose that there is a natural constraint such that smoky situations involve fiery situations: $T \Rightarrow T'$. Suppose that a situation s is a smoky situation, i.e., for some f , $s : T[f]$. If the constraint holds, then there is some fiery situation s' such that $s' : T'[f]$.

A reflexive principle of involvement is possible:

Definition 3.5 Reflexive Principle of Simple Involvement (Seligman 1990b, 153; Devlin 1994). If $s : T[f]$ for some anchor f and $T \Rightarrow T'$ then $s \models T'[f]$.

This principle entails the general (and weaker) principle of simple involvement. Furthermore, it is not particularly in line with the motivating idea of information flow (Seligman 1990b, 153). Nonetheless, clearly facts about a situation might carry information about itself, and one might want to distinguish this as a special case.

Although much neglected (Barwise 1989f, 274 fn. 9), situation theory has another sort of constraint that does not depend on a relation of involvement, namely *negative constraints*, or the relation of *preclusion*. Preclusion is a negative constraint that holds between two types such that if a situation is of the first type, then no compatible situation is of the second type⁴. Thus preclusion requires a different relation than the involvement relation, namely the relation of preclusion. If a situation type T precludes a situation type T' we write: $T \perp T'$.

³ This is a running example throughout the situation theoretic literature, e.g. (Barwise and Perry 1983) and (Devlin 1991).

⁴ Note that if we assume that every well-formed infon or its dual is supported by some situation, then we can model the relation of preclusion using the relation of involvement, namely by supposing that $T \perp [s \mid s \models \sigma]$ iff $T \Rightarrow [s \mid s \models \bar{\sigma}]$. A similar assumption is made in the discussion in (Barwise 1989f, 274).

Definition 3.6 Global Principle of Preclusion (Barwise and Perry 1983, 103-105; Seligman and Moss 2011, 317). If $s : T[f]$ and $T \perp T'$ then there is *no* compatible situation s' such that $s' : T'[f]$.

Remark. This is a rather strong principle (Seligman 1990b, 154). In some cases a weaker version of preclusion may be preferable. One may wish to locate preclusion to a single situation:

Definition 3.7 Local Principle of Preclusion (adapted from Seligman 1990b, 154). If $s : T[f]$ and $T \perp T'$ then $s \not: T'[f]$ ⁵.

Constraints are classified in a number of ways. For example, Seligman and Moss (2011, 315-317) include *necessary constraints*, of the sort that follow by definition or logical necessity, *conventional constraints*, constraints that hold by convention, as in norms of behavior and language, *nomie constraints*, constraints that hold by natural law, and *meta-theoretic constraints* of situation theory, *reflexive constraints*, those constraints that hold between two situations, *general constraints*, constraints that hold irrespective of which individuals are involved, and *preclusion*.

Example 3.2 (Lee, 2010). *Photinus pyralis* is a species of firefly endemic to the eastern United States. The mating ritual of the *P. pyralis* makes crucial use of its ability to communicate using light signals (Lloyd 1986, 113-114). Through brief sequences of flashes, the *pyralis* male indicates to any present female *P. pyralis* its willingness to mate. If a female *pyralis* receives this signal, it may

⁵ Seligman gives this definition in terms of a typing relation between situations and unstructured types. Seligman's definition, therefore, does not relativize to any anchor f , in keeping with his paper's contention that a priori individuation of the constituents of infons is premature.

respond with a half-second flash of its own. This process is repeated until the two *P. pyralis* locate and mate with each other.

The previous paragraph indicates types of situations involving fireflies. Let us model two of them as follows:

1. $T = [\dot{s} \mid \dot{s} \models \langle\langle IsSignal; \dot{x}, \dot{t}, \dot{l}; + \rangle\rangle \wedge \langle\langle HasPattern; \dot{x}, \mathbf{P}, \dot{t}, \dot{l}; + \rangle\rangle]$
2. $T' = [\dot{s} \mid \dot{s} \models \langle\langle IsSignaledBy; \dot{x}, \dot{y}, \dot{t}, \dot{l}; 1 \rangle\rangle \wedge \langle\langle IsInterestPyrFem; \dot{y}, \dot{t}, \dot{l}; 1 \rangle\rangle]$

We may posit a constraint $T \Rightarrow T'$ such that if in a situation s , there is a signal \mathbf{x} having pattern \mathbf{P} at some time \mathbf{t} and location \mathbf{l} , then there is some situation s' supporting the information that \mathbf{x} was signaled by a \mathbf{y} and \mathbf{y} is a female *pyralis* interested in mating at time \mathbf{t} and location \mathbf{l} . The propositional information carried is the proposition that there exists a situation s' that supports the information that \mathbf{y} is a female *pyralis* who signaled \mathbf{x} and is interested in a mating opportunity at \mathbf{t} and \mathbf{l} .

The Xerox principle says that information flow is necessarily reliable. However, this principle must be contrasted with the evident unreliability of information in the real world. Our firefly example is now particularly apt. As it turns out, the *Photinus pyralis* male is frequently the victim of the female members of the *Photuris* genus of fireflies. Members of this genus mimic the light signals of a female *Photinus pyralis* in order to lure males as their prey (Lloyd 1986, 15). The problem is thus: for philosophical reasons relating to the conditions for knowledge, information flow demands reliability. But processes that we commonly consider to carry information, like the one above, also fail to be reliable. Instead, situation theorists have sought some means to preserve the idea that the flow of information is reliable while simultaneously accommodating the existence of exceptions.

Conditional Constraints in Classic
Situation Theory

An early attempt at doing this within situation theory is through a theory of conditional constraints (Barwise and Perry 1983; Barwise 1989b). An *absolute* or *unconditional constraint* is a constraint $T \Rightarrow T'$ between two situation types of the sort already discussed. A conditional constraint is a constraint that holds only if some background condition B is satisfied, which may be represented as a ternary relation: $T \Rightarrow T'|B$ where the constraint $T \Rightarrow T'$ is conditioned on the compatible situation type B . In Barwise and Perry (1983) this ternary relation is reduced to the unconditional constraint: $B \cap T \Rightarrow T'$. Given a constraint $T \Rightarrow T'|B$, if a situation is both of type B and type T , relative to an anchor f , then there should exist some situation having type T' , relative to the anchor f . We may call this the *principle of conditional involvement*:

Definition 3.8 Principle of Conditional Involvement (Barwise and Perry 1983, 112-114; Barwise 1989b; Seligman and Moss 2011, 318). If $T \Rightarrow T'|B$ is a conditional constraint, then for every anchor f , if $s : B[f]$ and $s : T[f]$ then there is a compatible situation s' such that $s' : T'[f]$.

As before, we have a Xerox principle, suitably modified. Notice, that the background situation type is held constant.

Definition 3.9 Xerox Principle with Conditional Involvement (Barwise 1989b, 122; Seligman and Moss 2011, 318). If $T \Rightarrow T'|B$ and $T' \Rightarrow T''|B$ then $T \Rightarrow T''|B$.

Let us return to our firefly example. As we have described, it turns out that male members of the species *Photinus pyralis* are frequently predated upon by female members of the *Photuris* genus. Therefore, the optimistically posited

constraint $T \Rightarrow T'$ cannot hold quite as generally as we had supposed before. But we do not want to abandon the idea that the signal can carry the information that there is a female *P. pyralis* firefly interested in mating; that is, after all, teleologically speaking, the information that the signaling is intended to convey between male and female *pyralis*⁶. To salvage our constraint, we need to relativize this constraint to some background condition *B* which somehow precludes a mimicking predator from being responsible for the signal. Perhaps the constraint should be enriched to preclude predators, e.g.:

$$T \Rightarrow T' \mid [s \mid \dot{s} \models \langle\langle IsPredator; \dot{y}, \dot{t}, \dot{l}; - \rangle\rangle].$$

With this background condition, we may rest assured that if the background condition is in force, then whoever or whatever is responsible for the signaling of pattern *P* is not a predator. But are these sorts of background conditions enough to guarantee that the right sort of information flows? Unfortunately, no.

Under-specificity of background conditions. For example, some non-predator might be responsible for the misleading signal with pattern *P*. For example, a sneaky scientist, for her own nefarious, but impeccably scientific ends, might be interested in fooling a *P. pyralis* male into acting as if there were a female *pyralis* interested in mating present. In this case, the background condition of the conditional constraint is satisfied, but the constraint fails to behave properly.

Of course we might enrich our background situation type to preclude situations in which sneaky scientists are present:

⁶ If it were not so, then there would be no point in the predator signaling to the male fire-fly in the way it is doing.

$$B' = [\dot{s} \mid \dot{s} \models \langle\langle \text{IsPredator}; \dot{y}, \dot{t}, \dot{l}; - \rangle\rangle \wedge \langle\langle \text{IsSneakyScientist}; \dot{y}, \dot{t}, \dot{l}; - \rangle\rangle].$$

But then, it seems that there could still be any number of circumstances that might cause the constraint to fail and that would require the background condition to be enriched further. Exhaustively excluding every possible circumstance that might cause a constraint $T \Rightarrow T'$ to fail is not a palatable option. Having the type T' itself as the background condition or making the situation type

$[\dot{s} \mid \dot{s} \models \langle\langle \Rightarrow; T, T'; + \rangle\rangle]$ the background condition are neither of them useful solutions (Seligman 1990a, 166)⁷. Seligman remarks that:

If we were to continue the list of exceptions to ‘smoke means fire’ we would surely end up saying that smoke only means fire if there is a fire which produced the smoke. Anything short of this would allow the possibility of exceptional circumstances in which the law failed to apply. But the result is almost tautological. (150).

Disjunction problem. If it is plausible that a signal can carry items of incompatible information, relative to different background conditions, then a disjunction of these indicated contents would be more reliable. A signal with pattern **P** is a less reliable carrier of the information that there is a female *pyralis* interested in mating than it is of the disjunctive information that there is a female

⁷ We have neglected to mention the role that restricted parameters might play in specifying constraints. A background type might be replaced with a restricted situation parameter conditioned on the conditioning infons of the background type. Furthermore, the parameters in the conditioning infons of the types in the involves-relation might be restricted in some way to preclude some anchors assigning objects for which the constraint would fail to hold. We assert, however, that this approach does not gain us any additional advantage over the approach of background types generally. For example, in our running example, we might restrict the parameter \dot{y} in T' to female fireflies of the right species, but this does nothing to solve the problem. Alternately, we might restrict the parameter \dot{x} in the type T so that it is signaled by a female firefly of the right species, and even go so far as to restrict the parameter to those interested in mating, but then we are in exactly the same sorts of situation as we were when using background types.

pyralis interested in mating *or* that there is a hungry predator about. A signal with pattern **P** is a more reliable carrier of information the less specific that information is. This closely resembles the old *disjunction problem* (see Bremer and Cohnitz 2004, 135-145 for an introduction, also Braisby and Cooper 1996; Barwise and Seligman 1994). If a token of a goat can cause me to have a representation of a donkey, that is, a misrepresentation, then what privileges the token so that it is properly that of a goat rather than that of ‘a goat or donkey’, a more reliable interpretation? An adequate theory of information flow should be able to answer this question.

Under-specificity of Involvement Relation

The model of information flow by involvement, both conditional and unconditional, is inadequate in at least one other important respect. A simple or conditional constraint only tells us some situation of some type exists. This is called by Seligman and Moss (2011, 318) the *problem of specificity*. Suppose that we have a smoky type situation. Then, given that there is a constraint between smoky type situations and fiery situations, we know that there is a fiery type situation somewhere. But what we really want to be able to say is that we know that there is a fiery type situation nearby, presumably one that is somehow involved with the smoky situation. What we are **not** interested in are fiery situations from long, long ago, or from far, far away. Furthermore, there may always be some situation of the appropriate type somewhere. We have to look no further than our own sun to find a long-prevailing fiery situation that threatens to render any information that a smoky situation might carry superfluous.

Early Modifications to Theory of Information Flow

Situation theorists were well aware of many of these problems. An extended effort was made to find an alternative model of information flow, one that would both accommodate exceptions and take into account connections between situations. Here, we will describe many of the earlier attempts to flesh out a more robust and interesting theory of information flow.

Barwise's Background Situation Model of Conditional Involvement

Barwise (1989f, 274-276; see also Seligman 1990b, 167-169) tweaks the model of constraints of Barwise and Perry (1983) by using a background situation (rather than background situation type) and by making an explicit reference to a partial order on situations, along with several other differences both minor and substantive, including the inclusion of the much-neglected negative constraints, originally introduced in Barwise and Perry (1983). Note: Barwise gives his definitions in terms of infons rather than types.

Definition 3.10. Let w be a world, a maximal situation, and let $b \leq w$ be some situation that is part of w . A positive constraint $\langle\langle \Rightarrow; \bar{x}, \sigma, \tau; + \rangle\rangle$ is given as an infon where \bar{x} is a collection of parameters, σ and τ are parametric infons with parameters in \bar{x} . If $b \models \langle\langle \Rightarrow; \bar{x}, \sigma, \tau; + \rangle\rangle$ and $s \leq b$ then for every anchor $f : \bar{x} \rightarrow Obj(s)$ such that $s \models \sigma[f]$ (where $Obj(s)$ is the object set of s), there exists a situation $s' \leq w$ such that $s \models \tau[f]$. A negative constraint is given by the infon $\langle\langle \perp; \bar{x}, \sigma, \tau; + \rangle\rangle$. If $b \models \langle\langle \perp; \bar{x}, \sigma, \tau; + \rangle\rangle$ and if for some situation $s \leq b$ there is an anchor $f : \bar{x} \rightarrow Obj(s)$ such that $s \models \sigma[f]$ then there is no situation $s' \leq w$ such that $s \models \tau[f]$.

Remarks. This proposal recommends itself by avoiding the under-specificity of background types in grounding regularities⁸. Background conditions are concrete contexts, and so any temptation to define semi-tautologous conditioning background types is neatly averted. Still, the approach does not solve the problem of under-specific information flow. When a conditional constraint holds, all we learn is that some situation in the world, which need not even be part of the background situation b , is of some type. As we remarked earlier, in cases where it is common that some situation in the world is of some type, the carried information that some situation is of this type will be far less informative than we might desire.

In contrast the preclusion relation seems far too strong. Consider a specific game of chess that serves as a background situation b for the in-game situation s in which it is White's first move. White cannot move either Rook in the first move of the game, and so s 's being White's first move precludes White from moving its Rook. Obviously we do not want to preclude there being any situation in the world in which White moves its Rook. There are other games and other moves. So either this is not a relation of preclusion, as defined above, or the burden is shifted onto adequate constructions of the situation types involved so that the situations are appropriately "connected".

An alternative interpretation. The interpretation just given of Barwise's (1989f) brief formulation is reasonable; it is the interpretation taken by Jeremy Seligman in (1990b), for example, although Seligman's interpretation is in terms of (internally undifferentiated) types rather than infons. However, we might also

⁸ Once one considers that situations may have extent in time as well as space, we see that one has not necessarily committed to an overly specific condition on constraints. One may think of such background situations as 'epochs' during which a collection of regularities obtain.

interpret his text to mean that the range of the constraint is to be restricted to situations that are also part of the background situation b , so that if a situation s in b supports $\sigma[f]$ there is some situation in b , $\tau[f]$, supporting this type. In terms of types, for positive constraints $T \Rightarrow T' | b$ we would require that when $s : T$ for some situation $s \leq b$ there be a situation $s' \leq b$ having type T' . This would partially relieve the problem of under-specificity by restricting the class of situations to those in b ⁹. Should b be sufficiently small, s' may be unique in being of type T' . Similarly, for conditional negative constraints like $T \perp T' | b$ we may require that that when $s : T$ for some situation $s \leq b$, there is no situation $s' \leq b$ having type T' . This weakens, somewhat, the relation of preclusion. Problems of the kind just discussed in our chess example can be avoided if the background situation b is sufficiently small¹⁰. On the other hand, if the background situations are too small, it would seem that positive and negative constraints holding in b would hardly deserve to be called regularities¹¹. For any T and T' , b will decide whether there are any situations in b of type T or T' .

An interesting proposal that we have not seen in the literature¹² is a generalization of the preceding one. We might let a constraint be conditioned upon

⁹ We stated before that if we assume infon persistence and the existence of a maximal actual situation w , then a simple restatement of simple involvement is that for a constraint $T \Rightarrow T'$ with a situation s of type T , then w is of type T' relative to f . Under this proposal, a constraint would carry the additional information that b is of type T' relative to f .

¹⁰ As we will see, Seligman (1990b) weakens the preclusion relation so that it is local to a specific situation, so that if $T \perp T'$ and s has type T , then s does not have type T' . In terms of Barwise's proposal we might view this as the special case where $b = s$.

¹¹ There is a general concern here that bears consideration. Correlations in data can be found whenever we like simply by throwing out any data that is inconsistent with the correlations we would like to have. That is hardly a basis to inspire confidence. Yet, a scheme in which regularities are conditioned against arbitrarily chosen background situation does not appear any better than that, unless there is some further justification for the constraint.

¹² Actually, it bears some resemblance to early channel theory, as we'll see.

a pair of situations $\langle b, d \rangle$ such that if $T \Rightarrow T' \mid \langle b, d \rangle$ and a situation $s \leq b$ is of type T , there is a situation $s' \leq d$ having type T' (relative to an anchor f). Similarly, for $T \perp T' \mid \langle b, d \rangle$, situations of type T in b would preclude situations in d of having type T' , relative to an anchor f .

Although we do not have the space to review their paper here, for an interesting critique of these and other accounts of conditional constraints, with particular emphasis on issues relating to persistence and the disjunction problem, we refer the reader to Braisby and Cooper (1996).

Architectures of Information

We next look at Israel and Perry (1990, 1991), who attempt to model architectures of information. Israel and Perry avoid some of the problems we have just discussed by identifying informational connections between situation types through the objects and parameters that are part of the infons conditioning them.

Simple and relative constraints. David Israel and John Perry are interested in identifying the connections between propositions that permit information to flow. Their first move in this direction is in Israel and Perry (1990). Israel and Perry distinguish between *simple* and *relative constraints*. Simple constraints correspond to the sorts of simple (unconditional) constraints described already,¹³ and the information they carry is pure information in the nomenclature of Israel and Perry, as we have already described. Relative constraints involve three types T , T' , and T'' , where T involves T' relative to T'' , so that when a situation s has type T and a situation s'' has type T'' there exists some situation s' having type

¹³ Israel and Perry use the infonic notation for these constraints.

T' . We write a relative constraint C as $C = (T \Rightarrow_{T''} T')$. Relative constraints are said to carry *incremental information*.

We give their definitions of simple and relative information below. For these definitions, recall that $Cond(T)$ is the conditioning infon of the type. Also, we say that an infon σ is *factual* if there is some situation s such that $s \models \sigma$.

Definition 3.11 Incremental Information (Israel and Perry 1990, 9-10). If C is a relative constraint, then a factual infon σ carries the *incremental information that P* relative to C and to the factual infon σ' iff the following two conditions hold:

1. $C = (T \Rightarrow_{T''} T')$
2. For every anchor f such that $\sigma = Cond(T)[f]$ and $\sigma' = Cond(T'')[f]$, P is the proposition that $\exists s'(s' \models \exists a_1 \dots a_n Cond(T')[f])$.

We make these notions more concrete using an example.

Example 3.3 Pure and Incremental Information (Lee, 2010). We revisit our previous example of mating signals of the *Photinus pyralis*. Recall that we had a constraint $T \Rightarrow T'$ between situation types

$$T = [s \mid s \models \langle\langle IsSignal; \dot{x}, \dot{t}, \dot{l}; + \rangle\rangle \wedge \langle\langle HasPattern; \dot{x}, \mathbf{P}, \dot{t}, \dot{l}; + \rangle\rangle]$$

and

$$T' = [s \mid s \models \langle\langle IsSignaledBy; \dot{x}, \dot{y}, \dot{t}, \dot{l}; 1 \rangle\rangle \wedge \langle\langle IsInterestedPyrFem; \dot{y}, \dot{t}, \dot{l}; 1 \rangle\rangle].$$

We would like to say that the *fact* that a signal \mathbf{x} has pattern \mathbf{P} at some time \mathbf{t} and location \mathbf{l} *carries* the pure information that there exists some (possibly distinct) situation having some \mathbf{y} at time \mathbf{t} and location \mathbf{l} such that \mathbf{y} is the signaler of the signal \mathbf{x} and is a female *pyralis* interested in mating, relative to the constraint $T \Rightarrow T'$.

We may compare this with the *incremental information* given by a relative constraint. We reformulate our types as follows:

$$T = [\dot{s} \mid \dot{s} \models \langle\langle IsSignal; \dot{x}, \dot{t}, \dot{l}; + \rangle\rangle \wedge \langle\langle HasPattern; \dot{x}, \mathbf{P}, \dot{t}, \dot{l}; + \rangle\rangle],$$

$$T' = [\dot{s} \mid \dot{s} \models \langle\langle IsInterestedPyrFem; \dot{y}, \dot{t}, \dot{l}; + \rangle\rangle]$$

and

$$T'' = [\dot{s} \mid \dot{s} \models \langle\langle IsSignaledBy; \dot{x}, \dot{y}, \dot{t}, \dot{l}; + \rangle\rangle].$$

Suppose that there exists a relative constraint $T \Rightarrow_{T''} T'$. We would like to say that the *fact* that a signal \mathbf{x} has pattern \mathbf{P} at some time \mathbf{t} and location \mathbf{l} *carries* the *incremental information* that there exists some situation in which the signaler of \mathbf{x} is the female *pyralis* interested in mating. The fact that \mathbf{y} is the signaler is the connecting fact, linking the individual who is an interested *pyralis* female to the individual who signaled with pattern \mathbf{P} , the information we are interested in. We see how incremental information shifts the focus of the indicated proposition away from the connecting fact, and onto the individuals who are constituents to both the connecting fact and the indicated proposition.

Information architectures. Israel and Perry (1991) build upon their work in Israel and Perry (1990) to illuminate how architectural relations between information carriers of indicating facts either induce or reflect relations between their indicated informational contents.

In order to do justice to the notions they develop, we must commit a few words of explanation to the terminology Israel and Perry employ. Israel and Perry (1991) expand upon their notion of pure and incremental information of Israel and Perry (1990). In particular they develop more fully a notion of *reflexive information*. Consider the following informational report:

The fact that the smoke alarm is blaring carries the information that the kitchen
has caught on fire.

Israel and Perry identify several components of statements like this one (Israel and Perry 1991, 147). The fact that the smoke alarm is blaring is called the *indicating fact* or *signal*. The *indicating property* is the property of blaring. The *carrier* of the information may be the smoke alarm, or perhaps the sound waves the smoke alarm is making having the property of blaring. The *indicated incremental content* of the statement is, roughly, the proposition that the kitchen may have caught on fire¹⁴, the kitchen itself is the *subject*, and the *indicated property* is the property of being on fire. The indicating fact carries the information of the indicated incremental content relative to a constraint and a *connecting fact*. In this case, the constraint is, approximately, that in a situation where a smoke alarm is blaring, then there is a connected nearby situation in which something has caught on fire. A *connecting*

¹⁴ For expediency, we are allowing ourselves to relax from an explicitly situation theoretic explication.

fact links an indicating fact to the indicated fact in some manner. In this case, although it is rather implicit, the connecting fact is the fact that the smoke alarm is in the kitchen. The following informational report is similar to the previous one, except in this case, the indicated content is not incremental, it is reflexive:

The fact that the smoke alarm is blaring carries the information that the kitchen in which the smoke alarm is located has caught on fire.

The principle difference is that the indicated informational content has the information carrier as a constituent, and hence there is no connecting fact identifying the kitchen. Instead, the subject of the indicated informational content—the kitchen—is identified by its relationship to the information carrier—the smoke alarm—namely it is the kitchen with the smoke alarm that is on fire, not just any location. Thus, in more situation-theoretic terms, the indicated proposition that there is some situation having as its type that it may have caught on fire, is delimited to it being a situation with a constituent that is connected to the signal, in this case the smoke alarm.

However, Israel and Perry note that the connections between indicating facts and indicated propositions can be more sophisticated. They can involve what Israel and Perry call *architectural connections* and *architectural relations* (148-149). Specifically, Israel and Perry (1990) identify three information architectures: *coincidence architectures*, *combinative architectures*, and *flow architectures*.

In the coincident architecture, the architectural relationship between the indicating facts (or signals) *induces* a relationship between the subjects of their respective indicated propositions.

Example 3.4. Suppose that a museum is exhibiting pieces by Californian artists Ernest Briggs and Nathan Oliveira, and that two of their paintings, one from each, are hanging opposite the other. Also, suppose, as is common, that besides each painting is a small placard with the name of the painting and the artist. The following two informational reports utilize a reflexive mode of presentation:

The fact that the placard reads Ernest Briggs carries the information that the painting that is beside the placard reading Ernest Briggs was painted by Ernest
Briggs.

The fact that the placard reads Nathan Oliveira carries the information that the painting that is beside the placard reading Nathan Oliveira was painted by Nathan
Oliveira.

Since the two paintings are opposite of each other in a local physical space, we can utilize this fact to pick out the subject of each informational report's indicated proposition:

The fact that the placard reads Ernest Briggs carries the information that the painting opposite the painting besides the placard reading Nathan Oliveira was
painted by Ernest Briggs.

The fact that the placard reads Nathan Oliveira carries the information that the painting opposite the painting besides the placard reading Ernest Briggs was
painted by Nathan Oliveira.

Example 3.5. A second very apt example of a coincident information architecture is provided by Israel and Perry (1991, 149-151), which we suitably adapt here. When my daughter was an infant, a visit to the pediatrician would commence by measuring our daughter's height and weight by having us lay our daughter on a scale inset into a table which also had a ruler parallel to the infant so that the height and weight would be simultaneously measured. The weight was registered on a digital display. The device is structured in such a way that, under normal operating conditions, the infant who is affecting the weight scale is the infant who is being measured by the ruler. Hence, the relationship between the weight registered on the digital display and the height indicated on the ruler—the two indicating facts or signals—is one of identity, and this architectural relationship induces a relationship between their respective indicated propositions so that the following informational reports are intelligible:

The fact that the weight scale's digital display reads 10.9 carries the information that the infant whose height is being measured by the ruler is 10.9 kilograms.

The fact that the infant's head touches the mark reading 82 carries the information that the infant affecting the weight scale's digital display is 82 centimeters tall.

The second information architecture described by Israel and Perry is the combinative architecture. In contrast to the coincident architecture, it is characteristic that in a combinative architecture the architectural relation between indicating facts characteristically reflects the relationship between the subjects of their respective indicated contents.

Example 3.6. Consider a map of Poland at a scale of one centimeter for every 100 kilometers. Upwards and to the right approximately 2.6 centimeters from a circle and label reading *Krakow* is another circle and label reading *Warsaw*. The spatial arrangement between these two circles on the map reflect the geographical relationship between the cities of Krakow and Warsaw, respectively, namely that the city of Warsaw is approximately 260 kilometers northwest of the city of Krakow. Thus for example, we can say that the fact that the circle on the map is labeled Warsaw carries the information that the city 260 kilometers northwest of Krakow is the city of Warsaw.

Example 3.7. Israel and Perry's example (1991, 152-154) is that of a labeled folder in a doctor's office where all the various records of a patient are kept together. These various records are stored together in a labeled folder, as a practice, precisely because they pertain to the same individual, and therefore this practice acts as a constraint that if a folder is labeled with a name *Name*, then there is a patient to which *Name* refers and all of the documents in the folder have that patient as its subject matter (153). Thus, we might have that the fact that the blood tests a glucose level of over 150 mg/dl carries the information that the patient referred to by the label *Name* may have diabetes, relative to the constraint that a document in the labeled folder pertain to the individual referred to by the name on folder's label.

Israel and Perry write that the previous two architectures provide *architecturally coordinated information* (Israel and Perry 1991, 154). The final information architecture that Israel and Perry discuss is the flow architecture, which they say provides *architecturally mediated information* (154). The flow architecture is essentially the same as the information flow in situation theory we

have already discussed. It is however worthwhile to give Israel and Perry's fine-grained restatement of Dretske's Xerox principle:

If (i) there are architectural constraints c and architectural connections C such that s carries the architectural information that b is F , relative to c given C , and (ii) there are constraints c' and connecting facts C' such that the fact that b is F carries the information that Q relative to c' and C' , then there are constraints c'' and connecting facts C'' such that s carries the information that Q relative to c'' and C'' . (Israel and Perry 1991, 157).

Seligman's Theory of Perspectives

We now turn back to the theory of perspectives of Seligman (1990a; Seligman 1990b, 155). Rejecting the standard approach in situation theory of classifying situations by situation types constructed from some prior individuation of the world into objects, relations and properties (Seligman 1990a, 149-150), Seligman makes the typing of situations ontologically and epistemically prior to the individuation of objects, properties and relations, which he shows can be reconstructed from this much simpler and less demanding foundation. Hence, the types in his theory of perspectives are primitives, though the formalism he presents does not necessarily preclude the usual info-based types from figuring in a perspective.

A perspective is a classification of a part of the world from a point of view. The paradigmatic case of a perspective is the classification of a visual scene, but Seligman intends his theory of perspectives to be more general, and so takes perspectives to be arbitrary classifications of situations satisfying some ontological first principles. In particular, Seligman models a perspective as a structure containing a set of situations, a set of types, a classification relation, and two relations modeling positive and negative constraints, satisfying several structural properties. Negative constraints are necessary because the theory does not

presume any internal structure to its types that would distinguish negative from positive types (153).

Definition 3.12 Perspectives (Seligman 1990b, 155). A *perspective* is a structure $\langle Sit, Typ, :, \Rightarrow, \perp \rangle$ in which *Sit* is a collection of situations, called the *domain* of the perspective, *Typ* is a set of types classifying those situations, $: \subseteq Sit \times Typ$ is a classification relation, and \Rightarrow and \perp are two binary relations on *Typ*, the involves relation and the precludes relation, respectively, satisfying four additional structural properties governing these three relations for all $s \in Sit$ and $T \in typ$:

1. *Involvement*¹⁵. If $s : T$ and $T \Rightarrow T'$ then there exists a $s' \in Sit$ such that $s' : T'$.
2. *Xerox*. If $T \Rightarrow T'$ and $T' \Rightarrow T''$ then $T \Rightarrow T''$.
3. *Local preclusion*. If $s : T$ and $T \perp T'$ then $\neg(s : T')$.
4. *Mutual preclusion*. If $T \perp T'$ then $T' \perp T$.

The following extensional definitions will simplify our discussion.

Definition 3.13 (Seligman 1990b, 158). Let $P = \langle Sit, Typ, :, \Rightarrow, \perp \rangle$ be a perspective. For all $T, T' \in Typ$ and situations $s, s' \in Sit$, define:

1. *Constant Conjunction*. $T \succ T'$ iff $\forall s$ if $s : T$ then $\exists s' s' : T'$
2. *Local Inconsistency*. $T \dagger T'$ iff $\forall s$ if $s : T$ then $\neg(s : T')$.
3. *Co-groundedness*. $T \succ\succ T'$ iff $\forall s$ if $s : T$ then $s : T'$.
4. *Global Inconsistency*. $T \dagger\dagger T'$ iff $\forall s$ if $s : T$ then $\neg\exists s' s' : T'$.

¹⁵ In Seligman (1990b) this is called *facticity*. However, here we call it after the principle of involvement, the definition of which we have already given, for the sake of consistency.

Remark. We may restate the principle of involvement and the principle of local preclusion as follows (159):

1. *Involvement.* If $T \Rightarrow T'$ then $T \succ T'$.
2. *Local Preclusion.* If $T \perp T'$ then $T \nmid T'$.

Seligman also defines two related versions of the principles of preclusion and involvement, that of *strong involvement*¹⁶ which asserts that if $T \Rightarrow T'$ then $T \succ \succ T'$, and that of *global preclusion* which asserts that if $T \perp T'$ then $T \nmid \nmid T'$. Strong involvement implies involvement, and global preclusion implies local preclusion (157). Seligman favors the weaker forms of these principles in defining perspectives on the basis that it is best to presume least, and the fact that the weaker form of involvement allows information to flow between situations. Seligman also considers two optional structural properties that a theorist may require perspectives to satisfy. The first of these is that the \Rightarrow relation be reflexive¹⁷. Seligman finds some reasons why a perspective's \Rightarrow relation might not be reflexive (157)¹⁸. The second principle Seligman considers governs the interaction of \Rightarrow and \perp , which we call the *mixed Xerox principle*: if $T \Rightarrow T'$ and $T' \perp T''$, then $T \perp T''$. Seligman (158) suggests that the appeal of this principle relies on a more global sense of preclusion, though not exactly the one we introduced earlier, and which is not entirely consonant with the theoretical orientation of Seligman's perspectives, and situation theory in general. While both the first and second principle may be suitable or even necessary in some

¹⁶ Seligman calls this strong facticity. As before, we have adopted the term involvement for the sake of consistency.

¹⁷ If \Rightarrow is reflexive then for all types $T \in Typ$, $T \Rightarrow T$.

¹⁸ Seligman suggests that although it seems very natural to assume reflexivity, as it is a property of most consequence relations, and can be justified extensionally, it may be the case that from certain practical perspectives, e.g., of lower animals, reflexive constraints may be well motivated by neither necessity nor utility (Seligman 1990b, 157).

perspectives, they are clearly too strong in the general case¹⁹, and the principle, as stated, leads to a number of difficulties²⁰, some of which Seligman illustrates in

¹⁹ For example, Seligman (176) identifies classical propositional logic as a perspective that always satisfies reflexivity and the mixed Xerox principle. If for a model m and formula ϕ , $m : \phi$ (meaning that ϕ is true in m) and $\phi \Rightarrow \gamma$ (meaning ϕ logically entails γ) then $m : \gamma$. And if $\gamma \perp \phi$ then $m : \phi$.

²⁰ Although it is not central to our discussion, it is worthwhile considering some of these difficulties in more detail. Suppose that we have a positive constraint $T \Rightarrow T'$, a negative constraint $T' \perp T''$ and that for a $s \in Sit$, $s : T$. What can we infer?

- | | | |
|----|--------------------|----------------------------|
| 1. | $T \Rightarrow T'$ | Hyp. |
| 2. | $T' \perp T''$ | Hyp. |
| 3. | $s : T$ | Hyp. |
| 4. | $s' : T'$ | 1, 3 Princ. of Involvement |
| 5. | $\neg(s' : T'')$ | 2, 4 Princ. of Preclusion |
| 6. | $T \perp T''$ | 1, 2 Mixed Xerox Princ. |
| 7. | $\neg(s : T'')$ | 3, 6 Princ. of Preclusion |

But at this step we cannot infer that $s : T'$. We would be able to infer this if it is known that $s = s'$, but we do not know this and cannot assume it in general. In fact, our story suggests that it is otherwise. Note, again, that it is perfectly acceptable that $\neg(s : T'')$ without it being the case that $s : T'$.

If $s \neq s'$ then the fact that $\neg(s : T'')$ appears to be some kind of accident, since it does not seem to be motivated by any consideration except the mixed Xerox principle itself. On the other hand it is perfectly understandable why $\neg(s : T'')$ follows from $s = s'$: this is merely where the weaker principle of involvement satisfies the stronger principle of local involvement coupled with local preclusion. This is always the case in the classical logic perspective, for example. However, in the general case, we do not know, even in any particular application of the rule, whether $s = s'$. In many cases they will not be.

These difficulties are best explained by example. Suppose that I am sitting at my desk and I see a spot of light on my wall coming from a beam of light passing through my window. Because of the various properties of the light on my wall I know that it is emitted from a flashlight. We may describe two situations here: one situation in my room in which I see the spot of light on the wall, and a situation outside in which a flashlight is on shining light through my window. That these can also be considered to be both occurring in a single larger situation is beside the point, if we take partiality seriously. I would like to say that the spot of light carries the information that in some nearby situation (outside my window in fact) there is a flashlight whose batteries are charged and whose switch is in the engaged or *ON* position, e.g. by constraints to the effect that $LIGHT \Rightarrow ON$ and $LIGHT \Rightarrow CHARGED$. Furthermore, suppose that the switch's being in the *ON* position precludes its being in the *OFF* position, i.e., $ON \perp OFF$. We would like to say that the spot of light on the wall precludes the switch's being in the *OFF* position, e.g. and $LIGHT \perp OFF$. This is a reasonable inference, but it would be unfortunate if we were then forced to conclude that the situation at my desk is the situation outside my window. Note, again, that it is perfectly acceptable that $\neg(s : OFF)$ without it being the case that $s : ON$.

Another related desirable inference shifts the locus to the distal situation indicated by the relation of involvement, i.e. if $s : T$ carries the information that there exists a situation s' such that $s' : T'$, and T' precludes T'' then $\neg(s' : T'')$. One possible route to achieve this is to make the preclusion relation ternary: T

his paper, although the nature of these difficulties are not precisely delineated there.

As many of these principles (or structural properties) are already familiar to us, it is worthwhile reiterating the crucial difference between this and the standard theory of information flow by constraints: information flow is relative to a perspective, as are the constraints that govern the flow of information. One way to interpret this is that Seligman's theory of perspectives gives us a theory of conditional constraints and conditional information flow (Seligman 1990b, 165-168). In particular, we might take a perspective to be the background condition of a conditional constraint. An unconditional constraint is a conditional constraint whose conditioning perspective contains the set of all situations that are part of a world.

Given that two perspectives might classify the world differently, it is natural to wonder just what sorts of relationships perspectives have with one another. In particular, can we translate between perspectives?

You might recall our working example on perspectival situations, with Watson, Holmes, and Lestrade seated at a table arguing about whether the salt is to the left or to the right of the pepper and how, in our little whimsical narrative, Holmes (and later Watson) was able to shift their perspective to see the situation as other eyes might see it.

precludes T' in s . Thus for example, if $s : T$ and $T \perp^s T'$, then $\neg(s' : T')$. We may then restate the mixed Xerox principle as saying that if $T \Rightarrow T'$ and $T' \perp^s T''$ then $T \perp^s T''$. So for example, if $s : T$, $T \Rightarrow T'$, and $T' \perp^s T''$ then $T \perp^s T''$ and $\neg(s' : T'')$. This solution resembles in some ways the model of conditional information in (Barwise 1989f) that we have already discussed. Whether this is an entirely satisfactory solution has yet to be explored. In any case (Seligman 1990b) is non-committal with respect to the mixed Xerox principle.

Shifts in perspective may be modeled as maps between perspectives. However, not all kinds of maps are suitable. Some do not, in general, preserve structural properties like involvement or the Xerox principle. Other structure preserving maps may be too inflexible to model the kinds of perspective shifts that we might like.

Seligman arrives at a minimal definition of a perspective shift, which we will call here a *simple perspective shift*. A simple shift between perspectives is intended to guarantee that if for two types in the shift's domain a positive (or negative) constraint holds for the shifts of those two types, then the extensional relation of constant conjunction (or local inconsistency) holds between the two types in the domain. Notice that the definition of a perspective does not require that there be a positive or negative constraint for every pair of types for which constant conjunction or local inconsistency holds. Thus correlations between types may hold in a perspective, either accidentally or by some genuine regularity, but nonetheless not be recognized as such in the perspective.

Definition 3.14 Simple Perspective Shift (Seligman 1990b, 163). Let $P = \langle \text{Sit}_P, \text{Typ}_P, \cdot_P, \Rightarrow_P, \perp_P \rangle$ and $P' = \langle \text{Sit}_{P'}, \text{Typ}_{P'}, \cdot_{P'}, \Rightarrow_{P'}, \perp_{P'} \rangle$ be perspectives. A map $\rho : \text{Typ}_P \rightarrow \text{Typ}_{P'}$ is a *shift* between perspectives P and P' , written $\rho : P \rightsquigarrow P'$, iff for all $T, T' \in \text{Typ}_P$,

1. *Shifted Involvement*. If $\rho T \Rightarrow \rho T'$ then $T \succ T'$.
2. *Shifted Local Preclusion*. If $\rho T \perp \rho T'$ then $T \dagger T'$.

Remark. Simple shifts preserve the structural properties prerequisite to a structure being a perspective, i.e., $P\rho$ is a perspective, the *shifted perspective* of the perspective P (163).

Simple shifts do not generally compose to give shifts, since by the definition of a shift a constraint in the range of the shift need not be in its domain (Seligman 1990b, 163)²¹. Seligman (163-4) gives two ways to strengthen the definition to ensure that shifts compose to give shifts, defining what he calls *strong* and *weak* shifts respectively.

Definition 3.15 Strong and Weak Shifts (Seligman 1990b, 163-4). Let $P = \langle \text{Sit}_P, \text{Typ}_P, \cdot_P, \Rightarrow_P, \perp_P \rangle$ and $P' = \langle \text{Sit}_{P'}, \text{Typ}_{P'}, \cdot_{P'}, \Rightarrow_{P'}, \perp_{P'} \rangle$ be perspectives. A map $\rho : \text{Typ}_P \rightarrow \text{Typ}_{P'}$ is a *strong shift* between perspectives P and P' iff for all $T, T' \in \text{Typ}_P$,

1. If $\rho T \Rightarrow \rho T'$ then $T \Rightarrow T'$.
2. If $\rho T \perp \rho T'$ then $T \perp T'$.

A map $\rho : \text{Typ}_P \rightarrow \text{Typ}_{P'}$ is a *weak shift* between perspectives P and P' iff for all $T, T' \in \text{Typ}_P$,

1. If $\rho T \succ \rho T'$ then $T \succ T'$.
2. If $\rho T \dagger \rho T'$ then $T \dagger T'$.

Remark. As can be observed, a strong shift preserves shifts over composition by guaranteeing that constraints are anti-preserved (reflected) over shifts, while a weak shift preserves shift over composition by guaranteeing that the extensional relations are reflected (Seligman 1990b, 263).

²¹ This is easily demonstrated (Seligman 1990b, 163). Given three perspectives P, P' and P'' and shifts $\rho_1 : \text{Typ}_P \rightarrow \text{Typ}_{P'}$ and $\rho_2 : \text{Typ}_{P'} \rightarrow \text{Typ}_{P''}$, $\rho_2 \rho_1$ is a perspective shift iff $\rho_2 \rho_1 T \Rightarrow \rho_2 \rho_1 T'$ implies that $T \succ T'$. It is true that if $\rho_2 \rho_1 T \Rightarrow \rho_2 \rho_1 T'$ then $\rho_1 T \succ \rho_1 T'$, but this alone is not sufficient to guarantee that $\rho T \Rightarrow \rho T'$, which in turn is necessary to guarantee that $T \succ T'$.

We pause to illustrate perspectives and perspective shifts by considering two examples. We begin with the modeling of shifts between two classical propositional languages as perspectives as in Seligman (1990b, 176-7). We then return to Holmes, Watson, and Lestrade and their respective perspectives on the placement of the salt and pepper.

Example 3.8 Shifts between two propositional logics (Seligman 1990b, 176). Seligman shows that for a structure $\mathcal{L} = \langle M, S, :, \Rightarrow, \perp \rangle$, if S is a set of well-formed formula of a language of propositional logic, M is the set of possible truth valuations of the atomic formula in S , and C is the propositional calculus such that for all $m \in M$ and for all $\varphi, \gamma \in S$, $m : \varphi$ iff φ is true in m , $\varphi \Rightarrow \gamma$ iff $\{\varphi\} \vdash_C \gamma$ and $\varphi \perp \gamma$ iff $\{\varphi\} \vdash_C p \wedge \neg p$ for some propositional variable p , then \mathcal{L} is a perspective. \mathcal{L} satisfies strong involvement, Xerox, local preclusion, and mutual preclusion. Also, the semantics of the classical propositional logic perspective is equivalent to the extensional relation \succ of co-groundedness (176-7). See Seligman's paper for a more thorough description of propositional logic perspectives, in particular his discussion of the limitations of the classical logic perspective.

Let $\mathcal{L}_1 = \langle M_1, S_1, :, \Rightarrow_1, \perp_1 \rangle$ and $\mathcal{L}_2 = \langle M_2, S_2, :, \Rightarrow_2, \perp_2 \rangle$ be two classical logic perspectives. Suppose that there is a shift $\rho : \mathcal{L}_1 \succ \mathcal{L}_2$. Then for all $\varphi, \gamma \in S_1$, $\rho\varphi \Rightarrow_2 \rho\gamma$ iff $\{\rho\varphi\} \vdash_{C_2} \rho\gamma$ iff $\varphi \succ_1 \gamma$ iff for all $m \in M_1$ if $m : \varphi$ then $\exists m' \in M_1 m' : \gamma$ ²². We may be interested to know whether something more can be said about the relationship between φ and γ in \mathcal{L}_1 . The soundness and completeness of classical propositional logic mean that $\varphi \Rightarrow_1 \gamma$ iff $\varphi \succ_1 \gamma$ (177),

²² For readability we drop the subscript on the $:$ relation. Also, for brevity we leave out the precludes relation.

but the simple shift only yields us the considerably weaker fact that $\varphi \succ_1 \gamma$. Note that φ and γ can be any well-formed formula; however γ can be a logical contradiction only if φ is.

Example 3.9. Salt and Pepper. We return to Holmes, Watson, and Lestrade seated about the table discussing the relative locations of the salt and pepper. The reader will recall that we had defined a number of situation types, which we might take as primitive types that may be used to classify situations of this sort. These included types such as $[\dot{s} \mid \dot{s} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; + \rangle\rangle]$, $[\dot{s} \mid \dot{s} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; - \rangle\rangle]$, $[\dot{s} \mid \dot{s} \models \langle\langle \textit{LeftOf}; \mathbf{salt, pepper}; + \rangle\rangle]$, $[\dot{s} \mid \dot{s} \models \langle\langle \textit{LeftOf}; \mathbf{salt, pepper}; - \rangle\rangle]$, $[\dot{s} \mid \dot{s} \models \langle\langle \textit{InFrontOf}; \mathbf{salt, pepper}; + \rangle\rangle]$, and $[\dot{s} \mid \dot{s} \models \langle\langle \textit{InFrontOf}; \mathbf{salt, pepper}; - \rangle\rangle]$.

Let us define three perspectives, that of Holmes, Watson, and Lestrade, respectively, such that each uses the same set of types to classify situations: $H = \langle \textit{Sit}, \textit{Typ}, :, \Rightarrow, \perp \rangle$, $W = \langle \textit{Sit}, \textit{Typ}, :, \Rightarrow, \perp \rangle$, and $L = \langle \textit{Sit}, \textit{Typ}, :, \Rightarrow, \perp \rangle$ ²³. Within each perspective we will expect that types will be related to one another by positive and negative constraints in regular ways. For example, within the perspective of Watson (or Holmes, or Lestrade) we would expect negative constraints such as:

$$[\dot{s} \mid \dot{s} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; + \rangle\rangle] \perp [\dot{s} \mid \dot{s} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; - \rangle\rangle]$$

and

²³ We omit subscripts to simplify the presentation. Context should make it clear which perspectives are being referred to.

$$[\dot{s} \mid \dot{s} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; + \rangle\rangle] \perp [\dot{s} \mid \dot{s} \models \langle\langle \textit{LeftOf}; \mathbf{salt, pepper}; + \rangle\rangle],$$

and positive constraints such as:

$$[\dot{s} \mid \dot{s} \models \langle\langle \textit{LeftOf}; \mathbf{salt, pepper}; + \rangle\rangle] \Rightarrow [\dot{s} \mid \dot{s} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; - \rangle\rangle],$$

and

$$[\dot{s} \mid \dot{s} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; + \rangle\rangle] \Rightarrow [\dot{s} \mid \dot{s} \models \langle\langle \textit{LeftOf}; \mathbf{pepper, salt}; + \rangle\rangle],$$

but not constraints such as:

$$[\dot{s} \mid \dot{s} \models \langle\langle \textit{LeftOf}; \mathbf{salt, pepper}; - \rangle\rangle] \Rightarrow [\dot{s} \mid \dot{s} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; + \rangle\rangle]$$

since it is possible that the salt might be neither to the left nor to the right of the pepper, as in fact they are not for Lestrade, who sees them one in front of the other. The challenge then is to relate their respective viewpoints in some manner that captures the underlying uniformity between them. We might note, for example, that a person in Watson's position would necessarily classify the salt as being to the left of the pepper given that a person in Holmes's position would classify it as being to the right of the pepper. In our story, it is precisely this sort of perspective shift that allows Holmes' and Watson to come to an understanding²⁴. Let us simplify things somewhat by introducing a simple notation for a subset of the types discussed above according to the following definitions:

- $R =_{df} [\dot{s} \mid \dot{s} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; + \rangle\rangle]$
- $\bar{R} =_{df} [\dot{s} \mid \dot{s} \models \langle\langle \textit{RightOf}; \mathbf{salt, pepper}; - \rangle\rangle]$
- $L =_{df} [\dot{s} \mid \dot{s} \models \langle\langle \textit{LeftOf}; \mathbf{salt, pepper}; + \rangle\rangle]$

²⁴ However, as the infamously dull Lestrade is intended to suggest, such perspective shifts require a little imagination.

- $\bar{L} =_{df} [\dot{s} \mid \dot{s} \models \langle\langle \textit{LeftOf}; \textbf{salt, pepper}; - \rangle\rangle]$
- $B =_{df} [\dot{s} \mid \dot{s} \models \langle\langle \textit{Behind}; \textbf{salt, pepper}; + \rangle\rangle]$
- $\bar{B} =_{df} [\dot{s} \mid \dot{s} \models \langle\langle \textit{Behind}; \textbf{salt, pepper}; - \rangle\rangle]$
- $F =_{df} [\dot{s} \mid \dot{s} \models \langle\langle \textit{FrontOf}; \textbf{salt, pepper}; + \rangle\rangle]$
- $\bar{F} =_{df} [\dot{s} \mid \dot{s} \models \langle\langle \textit{FrontOf}; \textbf{salt, pepper}; - \rangle\rangle]$

Let us suppose that the perspective of Holmes, for whom the salt is to the right of the pepper, is as follows: for types, it has the types we have just described as well as others we have not perhaps considered; for situations it has the set of situations in which Holmes is seated just so during the discussion between Holmes, Watson, and Lestrade; and let the constraints be those we have already described²⁵. We designate this perspective as \mathcal{H} . Table 1 gives us a fragment of the classification from the perspective of Holmes in tabular format, where a 1 indicates that the situation has the type indicated in the column, and a 0 indicates that it does not have that type.

Note that every situation in this perspective is indistinguishable with respect to the types $R, \bar{R}, L, \bar{L}, B, \bar{B}, F$, and \bar{F} , although they may not be indistinguishable with respect to other types in the classification.

Further let us suppose that the perspectives of Watson who is seated opposite of Holmes and for whom the salt is to the left of the pepper, is in all respects just like that of Holmes except with regard to its set of situations and their classification. We designate this perspective \mathcal{W} . Likewise, the perspective \mathcal{L} of Lestrade, who is seated such that the salt is directly in front of the pepper, only differs in its set of situations and how those situations are classified. Table 2 gives

²⁵ We are free to include constraints unrealized by any situation in *Sit*. For example, no situation in Holmes's perspective has type L , despite $L \Rightarrow \bar{R}$ in his perspective, since it is only required that if $L \Rightarrow \bar{R}$ and if for some situation $s : L$, then $s' : \bar{R}$ for some situation s' ; but there is no situation with type L in Holmes's perspective.

a fragment of the classification from the perspective of Watson and Table 3 gives a fragment of the classification of Lestrade's perspective.

Table 1. Fragment of Classification from Perspective of Holmes, \mathcal{H}

Situations	Types								
	R	\bar{R}	L	\bar{L}	B	\bar{B}	F	\bar{F}	...
h_1	1	0	0	1	0	1	0	1	...
h_2	1	0	0	1	0	1	0	1	...
h_3	1	0	0	1	0	1	0	1	...
h_4	1	0	0	1	0	1	0	1	...
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\ddots

Table 2. Fragment of Classification from Perspective of Watson, \mathcal{W}

Situations	Types								
	R	\bar{R}	L	\bar{L}	B	\bar{B}	F	\bar{F}	...
h_1	0	1	1	0	0	1	0	1	...
h_2	0	1	1	0	0	1	0	1	...
h_3	0	1	1	0	0	1	0	1	...
h_4	0	1	1	0	0	1	0	1	...
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\ddots

Each of these three perspectives can be related to the other by perspective shifts. We demonstrate by defining an appropriate perspective shift between the

perspectives²⁶ of Holmes and Watson. Define a map $\rho : Typ_{\mathcal{H}} \rightarrow Typ_{\mathcal{W}}$ as follows:
 $\rho(R) = L$, $\rho(\bar{R}) = \bar{L}$, $\rho(L) = R$, $\rho(\bar{L}) = \bar{R}$, $\rho(B) = B$, $\rho(\bar{B}) = \bar{B}$, $\rho(F) = F$,
 and $\rho(\bar{F}) = \bar{F}$.

Table 3. Fragment of Classification from Perspective of Lestrade, \mathcal{L}

Situations	Types								
	R	\bar{R}	L	\bar{L}	B	\bar{B}	F	\bar{F}	...
l_1	0	1	0	1	0	1	1	0	...
l_2	0	1	0	1	0	1	1	0	...
l_3	0	1	0	1	0	1	1	0	...
l_4	0	1	0	1	0	1	1	0	...
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\ddots

Proposition 3.2. The map ρ is a strong perspective shift.

Proof. A partial discursive proof. A map $\rho : Typ_{\mathcal{H}} \rightarrow Typ_{\mathcal{W}}$ is a strong shift whenever if $\rho(\varphi) \Rightarrow \rho(\gamma)$ then $\varphi \Rightarrow \gamma$ and if $\rho(\varphi) \perp \rho(\gamma)$ then $\varphi \perp \gamma$. We demonstrate that this condition holds for several of the constraints we have considered so far:

- $\rho(L) \Rightarrow \rho(\bar{R})$ is a constraint of \mathcal{W} and $L \Rightarrow \bar{R}$ is a constraint of \mathcal{H} .
- $\rho(R) \Rightarrow \rho(\bar{L})$ is a constraint of \mathcal{W} and $R \Rightarrow \bar{L}$ is a constraint of \mathcal{H} .
- $\rho(F) \Rightarrow \rho(\bar{L})$ is a constraint of \mathcal{W} and $F \Rightarrow \bar{L}$ is a constraint of \mathcal{H} .
- $\rho(L) \perp \rho(R)$ is a constraint of \mathcal{W} and $F \perp \bar{L}$ is a constraint of \mathcal{H} .

²⁶ More accurately, we demonstrate a perspective shift between those fragments of the three perspectives that we have represented.

- $\rho(L) \perp \rho(\bar{L})$ is a constraint of \mathcal{W} and $L \perp \bar{L}$ is a constraint of \mathcal{H} .
- $\rho(F) \perp \rho(R)$ is a constraint of \mathcal{W} and $F \perp R$ is a constraint of \mathcal{H} .
- $\rho(F) \perp \rho(L)$ is a constraint of \mathcal{W} and $F \perp L$ is a constraint of \mathcal{H} .

We may define a strong perspective shift $\mathcal{W} \rightsquigarrow \mathcal{H}$ in a reciprocal fashion.

We may also identify a shift $\theta : \mathcal{H} \rightsquigarrow \mathcal{L}$ from the perspective of Holmes to that of Lestrade by the map $\theta : \text{Typ}_{\mathcal{H}} \rightarrow \text{Typ}_{\mathcal{L}}$ for which $\theta(L) = B$, $\theta(\bar{L}) = \bar{B}$, $\theta(R) = F$, $\theta(\bar{R}) = \bar{F}$, $\theta(F) = L$, $\theta(\bar{F}) = \bar{L}$, $\theta(B) = R$, and $\theta(\bar{B}) = \bar{R}$.

As a practical matter, in many applications only partial mappings between two perspectives may be found, although complete mappings might exist between certain closely related perspectives, for example those for which an appropriate type has been added to the range's type-set.

On the other hand, the structural properties of shifts are not sufficient to make those shifts meaningful. For example, for any perspective having only two types, φ and γ , in its type-set, there are really only a few possibilities: $\varphi \Rightarrow \gamma$, $\gamma \Rightarrow \varphi$, both $\varphi \Rightarrow \gamma$ and $\gamma \Rightarrow \varphi$, $\varphi \perp \gamma$, or φ and γ are independent. It is trivial to find two two-type perspectives with a strong perspective shift between them, even when there really is no connection between them: one could be a classification of rocks on Mars and the other a classification of fish in the sea. All that is necessary is that both perspectives have similar patterns of constraints²⁷.

²⁷ We give two examples. Suppose that we have a perspective with a classification of coin flips in which there is a constraint that Heads precludes Tails, and that in another perspective we have a classification of a light bulbs and a constraint that On precludes Off. Various a strong shift can map Heads to On and Tails to Off or Heads to Off and Tails to On (and the other way around). But why should one perspective be thought to have anything to do of substance with the other? Some sort of connection between them must be established.

For our second example, let us have two perspectives both having types T and T' such that in the first perspective there is a constraint that $T \Rightarrow T'$ but not $T' \Rightarrow T$, while the second perspective has the constraint that $T' \Rightarrow T$, but not $T \Rightarrow T'$. We can find a strong perspective shift between these by simply having the shift map T to T' and T' to T . This may or may not be a sensible perspective shift.

For reasons like these, Jeremy Seligman's theory of information channels, an outgrowth of his theory of perspectives, posits *connections* between situations that permit information flow. It is to early channel theory that we now turn.

Early Channel Theory

The notion of *channels* linking situations was first introduced in Seligman (1991). However, what has become known as early channel theory was mainly introduced in Barwise (1992) and Barwise (1993). See Mares, Seligman, and Restall (2011) for a useful introduction to early channel theory.

Early channel theory enriches the standard account of information flow in situation theory by adding objective directed links between situations, called channels, along which information flows. Channels support constraints and so stand to constraints as situations stand to infons or situation types (Mares, Seligman, and Restall 2011, 333). In channel theory, the background conditions on constraints are implicit, in that information flow is relativized to channels.

A channel may be viewed as a map between two sets of situations. When a channel c connects a situation s to a situation t we write $s \xrightarrow{c} t$. We say that c is a *signaling relation* from the *source* s to the *target* t . In general a channel may connect more than one pair of situations and situations may be connected by many different channels.

Definition 3.16 Soundness Axiom (Barwise 1993, 13). A channel c supports a constraint $T \Rightarrow T'$, written $c \models T \Rightarrow T'$, iff for all situations s and t , if $s \xrightarrow{c} t$ and $s : T$ then $t : T'$.

Consequence. Principle of Involvement for Early Channel Theory. If $c \models T \Rightarrow T'$, $s : T$, and $s \xrightarrow{c} t$ then $t : T'$.

Remark. It may be noted that no mention is made of anchors. From the development of early channel theory and onward, most mention of anchors was dropped. In part this reflects a shift in the theory from an emphasis on the structure of infons and abstracts to one which emphasized the relations between types. It also reflects a tendency to regard types as possibly being more general than the kind given by abstraction. Seligman's (1990a; 1990b) types are primitives, for example. However, no misunderstanding should occur, if we understand that if the types are parametric, then there needs to be some anchor.

Barwise (1992) offers four principles of information flow that he says should be respected by his channel theory (see also Mares, Seligman, and Restall 2011, 330-333). These include a form of the Xerox principle, a principle governing logical entailment, and two principles that govern how channels can be composed. For the latter two, we will need the following definitions²⁸.

Definition 3.17 Serial channel composition (Barwise 1993, 18). A channel $c = c_1; c_2$ is a serial composition of channels c_1 followed by c_2 if for all situations s_1 in the domain of c_1 and all situations s_2 in the range of c_2 , $s_1 \xrightarrow{c} s_2$ iff there is an s such that $s_1 \xrightarrow{c_1} s$ and $s \xrightarrow{c_2} s_2$.

Remark. An easily proven consequence of this definition (Barwise 1993, 18) and that of a channel is that given $c = c_1; c_2$ if $c_1 \models T \Rightarrow T'$ and $c_2 \models T' \Rightarrow T''$ then $c \models T \Rightarrow T''$. If, as Barwise does, we warrant that any two channels has a composition, in effect licensing vacuous channels, then the Xerox principle follows.

²⁸ These definitions are not intended to apply to the channels of later channel theory. The channels of later situation theory are different sorts of entities.

Definition 3.18 Parallel channel composition (Barwise 1993, 21). A channel $c = c_1 \parallel c_2$ is a parallel composition of channels c_1 and c_2 if for all situations s, t :

$$s \xrightarrow{c} t \text{ iff } s \xrightarrow{c_1} t \text{ and } s \xrightarrow{c_2} t.$$

Remark. Barwise (1993, 21-22) argues that in some applications the existence of a parallel composition of two channels may not be warranted. We will define the addition principle under the definition of a partial operation of parallel composition.

We are now ready to present the four principles of information flow that Barwise argues (early) channel theory ought to respect:

Definition 3.19 Principles of Information Flow in Early Channel Theory (Barwise 1993; Mares, Seligman, and Restall 2011, 332-333).

1. *Xerox Principle.* Given that $c_1 \models T \Rightarrow T'$, $c_2 \models T' \Rightarrow T''$, $s \xrightarrow{c_1} s'$, $s' \xrightarrow{c_2} s''$ then if $s : T$ then $s'' : T''$.
2. *Logic as Information Flow.* $T \vdash T'$ iff $1 \models T \Rightarrow T'$, where 1 is the identity channel mapping each situation onto itself.
3. *Information Addition.* Assume that $c_1 \parallel c_2$ exists. If $c_1 \models T_1 \Rightarrow T_1'$ and $c_2 \models T_2 \Rightarrow T_2'$ then $c_1 \parallel c_2 \models (T_1 \wedge T_2) \Rightarrow (T_1' \wedge T_2')$ so that for $s \xrightarrow{c_1 \parallel c_2} t$ if $s : T_1$ and $s : T_2$ then $t : T_1'$ and $t : T_2'$.
4. *Exhaustive Cases.* Assume that $c_2 \parallel c_3$ exists. If $c_1 \models T_1 \Rightarrow T_2 \vee T_2'$ and $c_2 \models T_2 \Rightarrow T_3$ and $c_2 \models T_2' \Rightarrow T_3$ then $c_1 ; (c_2 \parallel c_3) \models T_1 \Rightarrow T_3$ so that for $s \xrightarrow{c_1 ; (c_2 \parallel c_3)} t$ if $s : T_1$ then $t : T_3$.

Remark. The principle of logic as information flow means that T entails T' whenever $s : T$ implies that $s : T'$. This is a local version of constraint.

Channels eliminate the problem of under-specificity of information flow, since information flows from one situation to another only through a channel, and the source situation and the channel together determine the target situation to which information flows. Despite this specificity, information agents may not be cognizant of the channel involved, and so may be in a position of only knowing a particular constraint is supported by some channel, but not whether the present situation is connected by a channel supporting that constraint. Thus in this picture, information flow is, roughly, veridical, but does not preclude the “flow” of pseudo-information. Likewise, the so-called problem of disjunction is arguably now a less pressing concern (Barwise and Seligman 1994, 349), as exceptions to constraints are now accommodated: a channel connecting the signaling situation to the target situation must actually support the constraint.

We illustrate these ideas now by recalling our running example of the firefly and its predatory mimic.

Example 3.10. Let us define as type T the type of situation in which a male firefly of the *Photinus pyralis* species intercepts a pattern of light of the kind. Situations of type T may involve situations in which there is female *pyralis* interested in mating, a situation of type M , i.e., there is a constraint $T \Rightarrow M$. But situations of type T may also involve situations in which there is a hungry predatory mimic interested in luring a male firefly closer, a situation of type pattern- P , i.e., there is a constraint $T \Rightarrow P$. Working on the assumption that a situation being of type pattern- P precludes it from being of type M , and vice-versa, $T \Rightarrow M$ and $T \Rightarrow P$ are incompatible constraints that cannot be supported by the same channel. Let us suppose further that if two channels have the same situation

in their domain then only one of these two constraints can be supported by either of the channels²⁹.

Let us suppose that in a particular situation s , a male firefly intercepts a signal of this kind, and so $s : T$. There are numerous possibilities. It might be that s is in the domain of a channel c_m supporting the constraint $T \Rightarrow M$ or it might be in the domain of a channel c_p supporting the constraint $T \Rightarrow P$, or s might be in the domain of neither channel. It might be in the sneaky-scientist channel, for example. It is doubtful that the male firefly is always in a position to discern which constraints are being supported. Suppose that s is in the domain of c_p ; a firefly mimic is responsible for the situation s 's being of type T . Then the situation s is a *pseudo-signal* for the firefly mating channel. We give a definition for a pseudo-signal below.

Definition 3.20 (Barwise 1993, 17). If $c \models T \Rightarrow T'$ and some situation $s : T$ but there is no situation t such that $s \xrightarrow{c} t$, then s is a *pseudo-signal* for $c \models T \Rightarrow T'$.

Therefore, early channel theory is able to explain at least one kind of possible error. Jon Barwise and Jeremy Seligman however become convinced that imperfect information flow involves types of error or exceptions not explainable as pseudo-signals, as we will see.

Disjunction Problem Again

We said earlier that channel theory suggests a solution to the disjunction problem. As Barwise and Seligman (1994, 349) point out, however, another form of the disjunction problem appears. This is easily demonstrated using our example. Let us suppose that in addition to our two types M and P we have a type

²⁹ We justify this by supposing that the same signal could not have been produced by two different sources.

$Z = M \vee P$, and we have a third channel $c_{m \vee p}$ supporting the constraint $T \Rightarrow Z$. It is quite reasonable to suppose that the situation s is in the domain of $c_{m \vee p}$ iff s is in the domain of c_m or s is in the domain of c_p , in which case if we are to take the situation s 's being T to mean one of these things only, then perhaps it should mean Z , since the constraint $T \Rightarrow Z$ holds more widely than either $T \Rightarrow M$ or $T \Rightarrow P$. Barwise and Seligman (1994, 349-350) argue that channels like $c_{m \vee p}$ and the constraints they support are less *natural*, and so not as preferable.

Applications of and Elaborations Upon Early Channel Theory

Applications to Relevance Logic. A number of scholars were quick to respond to early channel theory's innovations. In particular, several scholars saw in early channel theory similarities to relevance logic (also called relevant logic). Relevance logics are non-standard logics designed to avoid some of the paradoxes of strict and material implication, and other failures of relevance of antecedents to consequents. There are a variety of proof-systems for relevance logics, but most of them involve the labeling of hypotheses introduced into proofs; and the required use of those labels in the proofs. For example, in Anderson and Belnap's proof system R, we would have:

1. $A_{\{1\}}$ *Hyp*
2. $A \rightarrow B_{\{2\}}$ *Hyp*
3. $B_{\{1,2\}}$ $1, 2 \rightarrow E$

In a standard possible-worlds framework, a conditional $p \rightarrow q$ is true at a world w iff for each world w' accessible from w , either p is false or q is true in w' . Which worlds are accessible is given by a binary accessibility relation R . The semantics of relevance logics are interesting. The Routley-Meyer semantics of

relevance logic uses a ternary accessibility relation. An implication $p \rightarrow q$ is true in a world w iff for all worlds u and v such that $Rwuv$, either p is false at u or q is true at v . We might rewrite the semantics of the relevant conditional in more channel-theoretic notation: $w \models p \rightarrow q$ iff for all u and v , if $u \xrightarrow{w} v$ and $u \models p$ then $v \models q$.

Restall (1996) observes that if we equate channels with situations, then the Soundness Axiom of early channel theory is equivalent to the Routley-Meyer semantics of the conditional. Once this identification of situations and channels is made, one can establish the following relationship:

$$s \xrightarrow{c} t \text{ iff } s; c \sqsubseteq t$$

where $;$ is the channel composition operation, and \sqsubseteq is a partial order on situations.

Restall puts this work to an analysis of certain anomalies of the conditional for which relevance logic has had trouble accounting. For example, Restall explains away certain failures of transitivity as pseudo-signals.

For additional commentary on the connections between channel theory and relevance logic, consult Bremer and Cohnitz (2004) and Mares, Seligman, and Restall (2011).

Sequent calculi for early channel theory. Barwise, Gabby and Hartonas (1994; see also 1996) introduce several related sequent calculi for early channel theory (also applicable to labeled deduction systems). The shape of each calculus reflects decisions or assumptions made about the structure of signaling relations between situations (called sites in their paper), choices concerning the structure of logical formulas, e.g. having a single- or two-sorted language, and whether the

logic will be formulated at the level of types or will be formulated in a way that makes the roles of situations and channels in the calculus explicit. We will describe the general properties characterizing and distinguishing these calculi.

In each calculus, the language of types is inductively constructed from a set of atomic expressions and four connectives using a simple context-free grammar. Two grammars are given, one for a single-sorted language L , and the other for a two-sorted language L^2 distinguishing situations and channels. For L^2 , the grammar is:

$$\begin{aligned} Exp_s &:= AtExp_s \mid (Exp_s \leftarrow Exp_c) \mid (Exp_s \downarrow Exp_c) \\ Exp_c &:= AtExp_c \mid (Exp_s \rightarrow Exp_s) \mid (Exp_c \circ Exp_c) \end{aligned}$$

where situations are classified by expressions of sort Exp_s and channels are classified by expressions of sort Exp_c . The single-sorted language L is recursively built up from the following grammar:

$$Exp := AtExp \mid (Exp \rightarrow Exp) \mid (Exp \leftarrow Exp) \mid (Exp \circ Exp) \mid (Exp \downarrow Exp)$$

If we define a single-sorted language L whose set of atomic expressions is the union of the atomic s -expressions and c -expressions of L^2 , then L^2 is a proper sub-language of L , since not every compound expression of L will respect the sortal restrictions of L^2 (Barwise, Gabby and Hartonas 1994, 11). The best choice of single-sorted or two-sorted language will depend upon the underlying structure of situations, channels and signaling relations of the information domain being modeled and reasoned about. The models of their calculi involve what they call an information network:

Definition 3.21 Information network (Barwise, Gabby and Hartonas 1994, 3). An *information network* is a tuple $\mathcal{N} = \langle S, C, \mapsto, ; \rangle$ where S is a set of situations, C is a set of channels, $\mapsto \subseteq S \times C \times S$ is the ternary signaling relation, and $;$ is a binary associative operation on C^{30} such that \mapsto and $;$ must satisfy the condition that for all channels c and c' :

$$\forall s, t [s \overset{a;b}{\mapsto} t \text{ iff } \exists r (s \overset{a}{\mapsto} r \text{ and } r \overset{b}{\mapsto} t)].$$

A model $\mathcal{M} = \langle \mathcal{N}, f \rangle$ is a pair where \mathcal{N} is an information network and f is a mapping from well-formed formulas to sets of situations or channels, respecting sortal restrictions if any.

Remark. The relations \mapsto and $;$ are respectively just the usual signaling relation of early channel theory and the serial channel composition as given in (Barwise 1993). Note that it is not required that the sets C and S be distinct.

The \circ and \downarrow connectives may be thought of as left and right composition or conjunction operations, while the connectives \rightarrow and \leftarrow may be thought of as left and right conditionals. The formulas of these languages have interpretations given in the following definition.

Definition 3.22 (Barwise, Gabby and Hartonas 1994, 11). For a given information model $\mathcal{M} = \langle \mathcal{N}, f \rangle$, define the *of-type* relation \models by:

1. for each atomic type A and $s \in S \cup C$, $s \models_{\mathcal{M}} A$ iff $s \in f(A)$
2. $c \models_{\mathcal{M}} (A \rightarrow B)$ iff $\forall s, t$ (if $s \models_{\mathcal{M}} A$ and $s \overset{c}{\mapsto} t$ then $t \models_{\mathcal{M}} B$)
3. $c \models_{\mathcal{M}} (A \circ B)$ iff $\exists c_1, c_2$ ($c_1 \models_{\mathcal{M}} A, c_2 \models_{\mathcal{M}} B$, and $c = c_1 ; c_2$)

³⁰ The authors use \circ as the composition operator. However, we will try to maintain consistency with previous notation for serial composition, and avoid confusion between the \circ connectives on types.

4. $t \models_{\mathcal{M}}(A \downarrow C)$ iff $\exists s, c (s \models_{\mathcal{M}} A, c \models_{\mathcal{M}} C, \text{ and } s \xrightarrow{c} t)$, and
5. $s \models_{\mathcal{M}}(A \leftarrow C)$ iff $\forall c, t (\text{if } c \models_{\mathcal{M}} C \text{ and } s \xrightarrow{c} t \text{ then } t \models_{\mathcal{M}} A)$.

At this point (Barwise, Gabby and Hartonas 1994) develop several Gentzen sequent calculi that they call $\mathcal{G}_\alpha, \mathcal{G}_\beta, \mathcal{G}_\gamma$, and \mathcal{G}_δ respectively. These differ from one another along a number of dimensions. The Gentzen calculi \mathcal{G}_α and \mathcal{G}_β take the basic units of information to be the type formulas of either L^2 in the case of \mathcal{G}_α or the single-sorted language L in the case of \mathcal{G}_β (Barwise, Gabby and Hartonas 1994,). The Gentzen calculi \mathcal{G}_γ and \mathcal{G}_δ take propositions of the form $(t : A)$ to be the primary units of information in the calculus, where A is a formula of the single-sorted language L , and t is either a situation or channel. \mathcal{G}_γ is the logic of a fixed information network \mathcal{N} permitting infinitary expressions and rules of inference, while \mathcal{G}_δ is a less-powerful logic over models of arbitrary information networks with the usual finitary rules of inference (Barwise, Gabby and Hartonas 1994, 24).

For the Gentzen calculi \mathcal{G}_α and \mathcal{G}_β , a sequent is a pair³¹ consisting of a finite non-empty *sequence* of types Γ and a formula T of either L^2 or L respectively. Two kinds of sequents are recognized in each (Barwise, Gabby and Hartonas 1994, 14, 20). An *s*-sequent $A, C_1, \dots, C_n \vdash B$ is valid in a model \mathcal{M} iff for every sequence $s_1 \xrightarrow{c_1} t_1 \xrightarrow{c_2} t_2 \xrightarrow{\dots} t_n$, if $s \models A$ and for each i $c_i \models C_i$ then $t_n \models B$. A *c*-sequent $C_1, \dots, C_n \Vdash C$ is valid in a model \mathcal{M} iff for every sequence c_1, \dots, c_n (of channels), if $c_i \models C_i$ for each i , then $c_1; \dots; c_n \models C$ ³².

³¹ In the case of the \mathcal{G}_α , additional sortal conditions must be observed by these pairs to be well-formed sequents.

³² In \mathcal{G}_α there are sortal criteria defining well-formed *s*-sequents and *c*-sequents, and so the distinction between \vdash and \Vdash is unnecessary, except as a notational convenience.

We refer the reader to Barwise, Gabby and Hartonas (1994, 1996) for the specific rules of the Gentzen calculi \mathcal{G}_α and \mathcal{G}_β . We restrict ourselves here to a few remarks about the two calculi as a whole.

The authors' most fundamental results about \mathcal{G}_α and \mathcal{G}_β are that they are decidable, sound and complete. They are also cut-free. Also of interest (Barwise, Gabby and Hartonas 1994, 16) is the relation the connectives have to the familiar logical connectives of conjunction and implication; the systems \mathcal{G}_α and \mathcal{G}_β are *substructural*, i.e., they do not include the usual structural rules of permutation, contraction, and weakening. If these structural rules were added to the calculus then the connectives \circ and \downarrow would devolve to the conjunction rule of the usual Gentzen calculus and the rules for the connectives \rightarrow and \leftarrow would become the usual rule for the material conditional.

The sequent calculus \mathcal{G}_β is closely related to the Lambek calculus (Lambek 1958), an extension of traditional categorial grammars. Categorial grammars are type-based grammars expressively equivalent to context-free grammars used to describe natural-language syntax. Like those in their paper, categorial grammars have rules for function application from right to left and from left to right. The Lambek calculus, also expressively equivalent to a context-free grammar, extends categorial grammar by adding a type-concatenation operator and additional deduction rules.

Barwise, Gabby and Hartonas (1996, 50) define a *Lambek information network* as an information network where $S = C$ and $s \xrightarrow{c} t$ iff $s; c = t$ (1996, 50). String concatenation is a simple example of such a network (1996, 51). For example, we can let the string 'Hello' be the source, the channel be the string 'world!' and the target the string 'Hello world!'. A *Lambek theory* is the smallest

set of sequents such that for every expression A and B of the language L , the sequents

$$A \downarrow B \vdash A \circ B$$

and

$$A \circ B \vdash A \downarrow B$$

are in the theory (Barwise, Gabby and Hartonas 1996, 57). Hence, in a Lambek theory, these two connectives are equivalent. In proving the completeness of their calculus they define a model \mathcal{M} to be a *characteristic model of a theory T* if all valid sequents in \mathcal{M} are provable from T (Barwise, Gabby and Hartonas 1996, 58). From the soundness of the calculus it also follows that every sequent provable from a theory is valid in its characteristic model, and so every theory has a characteristic model; the completeness of the calculus immediately follows (59). They show that every extension of a Lambek theory has a characteristic model whose network is a Lambek network (Barwise, Gabby and Hartonas 1996, 59).

There are some interesting similarities here with the integration of early channel theory and relevant logic in Restall (1996). Like in the channel-theoretic Lambek calculus \mathcal{G}_β , Restall collapses the distinction between channels and situations, which Mares, Seligman, and Restall (2011, 333) describe as a kind of “flattening” of the channel-theoretic account. Restall also defines signaling relations in terms of serial composition: $s \xrightarrow{c} t$ iff $s; c \sqsubseteq t$ where \sqsubseteq is a partial order on situations. Restall justifies this relationship with the interpretation that, when the information in c is applied to information in s , it gives no more than

what information is already in t (Restall 1996, 467). Finally, it is perhaps worth noting that Barwise, Gabby and Hartonas (1994, 51) end their technical report with a call for an investigation into the relationship between relevant logic and their calculi, noting the similarities between the two and observing that while the development of relevant logic's inference system preceded the development of an adequate semantics for it, channel theory has worked in the opposite direction, working from intuitions about what a semantics of information flow should look like to a system of formal inference respecting those intuitions.

The Gentzen calculi \mathcal{G}_γ and \mathcal{G}_δ are also both sound and complete. Although both are based upon the single-sorted language L , the primary units of the second calculus are constructions of the form $[s : T]$, where s is a situation or channel and T is a type-formula of L . A construction of this form is treated as an atomic proposition. Compound propositions are constructed using the familiar logical connectives and quantifiers of logic.

More specifically (Barwise, Gabby and Hartonas 1994, 24-26), in \mathcal{G}_γ the class of propositions is the closure of the set of atomic propositions under negation and infinite conjunction. Infinite disjunctions and the conditional are defined in terms of negation and conjunction in the expected way. Given a fixed network \mathcal{N} and a model $\mathcal{M} = \langle \mathcal{N}, f \rangle$, a proposition $[s : T]$ is true in \mathcal{M} , written $\mathcal{M} \models [s : T]$, if $s \vDash_{\mathcal{M}} T$. A sequent $\langle \Gamma, \Delta \rangle$ is a finite pair of sets of propositions and is called \mathcal{N} -valid if in any model when every proposition in Γ is true, then some proposition in Δ is true.

In lieu of a complete description of all the inference rules for \mathcal{G}_γ , we offer the following example of an inference rule from their system.

Example 3.11. As a simple example consider the following inference rule of \mathcal{G}_γ .

$$\frac{\text{For every } c \text{ and } t \text{ s.t. } s \xrightarrow{c} t, \Gamma, [c : C] \vdash \Delta, [t : A]}{\Gamma \vdash \Delta, [s : (A \leftarrow C)]}$$

Observe that the quantifiers occur in the meta-language and not the object language, and that in order to apply the rule we have to show that $\Gamma, [c : C] \vdash \Delta, [t : A]$ is true for every c and t for which $s \xrightarrow{c} t$ in the network \mathcal{N} (Barwise, Gabby and Hartonas 1994, 28).

The calculus \mathcal{G}_δ for arbitrary networks is a little different. It introduces an infinite set of variables and a function symbol \circ to inductively construct a set of terms, and two additional symbols \mapsto and $=$, out of which atomic formulas may be constructed, of which there are three basic kinds: $t_1 \xrightarrow{t_2} t_3$, $[t : A]$, and $t_1 = t_2$, where t, t_1, t_2 and t_3 are terms. The set of formulas is closed under negation, finite conjunction, and universal quantification. Disjunction, implication, equivalence, and existential implication may be defined in terms of these. The semantics are somewhat more complicated than before and are defined as follows. In brief, let g be a function mapping variables to elements of a model \mathcal{M} . A formula p is satisfied by g in a model \mathcal{M} as follows. In addition to the usual rules for negation, conjunction and quantification, we have:

$$\mathcal{M} \models [t : A][g] \text{ iff } g(t) \models_{\mathcal{M}} A,$$

$$\mathcal{M} \models (t_1 \xrightarrow{t_2} t_3)[g] \text{ iff } g(t_1) \xrightarrow{g(t_2)} g(t_3),$$

and

$$\mathcal{M} \models (t_1 = t_2)[g] \text{ iff } g(t_1) = g(t_2).$$

A sequent is a pair of finite sets of formulas of our language. A sequent $\Gamma \vdash \Delta$ holds in a model \mathcal{M} under an assignment g if and only if $\mathcal{M} \models p[g]$ holds for every $p \in \Gamma$ only if $\mathcal{M} \models q[g]$ for some $q \in \Delta$, and is valid if the sequent holds under every assignment. As before we ask our reader to consult their paper for the exact listing of inference rules for this calculus, but give the following as representative examples.

Example 3.12. In the following two inference rules, Γ and Δ are sets of formulas, and s, c, d, r , and t are arbitrary terms, and x and y are arbitrary variables. The first determines how the signaling relation behaves under channel composition. The second stipulates that anything that is provable under the assumption that $\Gamma, x \overset{y}{\mapsto} t, [x : A]$, and $[y : C]$ is also provable under Γ and $[t : (A \downarrow C)]$, which is easily seen to accord with the meaning of formulas of the form $(A \downarrow C)$.

$$\frac{\Gamma \vdash \Delta, s \overset{c}{\mapsto} r \quad \Gamma \vdash \Delta, r \overset{d}{\mapsto} t}{\Gamma \vdash \Delta, s \overset{c \circ d}{\mapsto} r}$$

$$\frac{\Gamma, x \overset{y}{\mapsto} t, [x : A], [y : C] \vdash \Delta}{\Gamma, [t : (A \downarrow C)] \vdash \Delta}$$

The Gentzen calculi we have discussed are calculi for perfect information flows. Central to channel theory's project was the development of a theory of information

flow robust enough to admit exceptions to constraints, without resorting to the techniques of background conditions we described before. We turn next to some of the subsequent developments in channel theory enabling channel theory to model imperfect information flow more robustly.

Intermediate Channel Theory

At the end of his paper *Constraints, channels and the flow of information*, John Barwise writes:

Since this paper was written, Jerry Seligman and I have written another paper on the same topic...As a result of this collaboration [with Jerry Seligman], we have come to consider the treatment of conditional constraints and exceptions to constraints presented in this paper inadequate....” (1993, 26).

Their collaboration resulted in the publication of two papers. Barwise and Seligman (1994) is an effort to give a philosophical explanation of their channel-theoretic treatment of imperfect information flows and fallible natural regularities. Barwise and Seligman (1993) go into more mathematical detail of their emerging theory. We designate the channel theory of this period intermediate channel theory as it contains aspects common to early channel theory and later channel theory, and aspects common to neither. In addition to these papers, Lawrence Cavendon (1995) cites two unpublished manuscripts of theirs from that period, sketching out a more complete picture of their ideas at the time. These include a set of Jeremy Seligman’s lecture notes (Seligman 1993) and what is presumably an early manuscript of their book (Seligman and Barwise 1993). Cavendon makes extensive use of intermediate channel theory in Cavendon (1995, 1996, 1998). We will avail ourselves of Cavendon’s relatively more complete descriptions of intermediate channel theory in what follows.

As we have already noted, early channel theory was capable of accounting for failures of information flow due to pseudo-signals, but unable to account for other sorts of errors and exceptions. Using ideas originally developed in Seligman (1990a, 1990b, and 1991), Barwise and Seligman modify their theory of channels to allow for exceptions arising when the normal circumstances that usually permit certain inferences to be safely made fail to obtain, such as would be the case if when I see the engine of my automobile running I mistakenly infer that my key must have been inserted in the ignition, as would normally be the case, but is not the case on this particular occasion³³. Roughly, a channel is modeled as a system of classifications connected by type-level and token-level homomorphisms. The *core* of the channel is a classification whose tokens are connections and whose types are constraints. Instead of a channel supporting a constraint, the connections of the channel classification are classified by the constraints they support. In this way, Barwise and Seligman are able to account for circumstances in which a constraint is part of a channel, but in which it fails to hold of a particular signal-target pair even when the signaling situation is of the right type. Note that this means that the Soundness Axiom and the Principle of Involvement of Early Channel Theory are no longer valid.

The notion of a classification employed diverges somewhat from that of Seligman (1990a; 1990b) by introducing both positive and negative classification relations. The particulars of a classification are variously called particulars, situations, sites, or tokens. We shall adopt the word token, as Barwise, Seligman, Cavedon, and others do. The choice is intended, in part, to reflect channel theory's greater generality and formal independence from situation theory.

³³ This might happen in a variety of relatively unusual but plausible circumstances; readers may make up their own stories.

Definition 3.23 Classification of Intermediate Channel Theory (Barwise and Seligman 1993, 256; Cavedon 1995, 33). A *classification*

$A = \langle tok(A), typ(A), k \rangle$ is a structure consisting of a set of tokens $tok(A)$, a set of types $typ(A)$, and partial function $k : tok(A) \times typ(A) \rightarrow \{+, -\}$. For any token $a \in tok(A)$ and type $\alpha \in typ(A)$, a is of *positive type* α , written $a :^+ \alpha$, iff $k(a, \alpha) = +$ and is of *negative type* α , written $a :^- \alpha$, iff $k(a, \alpha) = -$.

Remark. The given definition permits tokens to be typed neither positively nor negatively by some type in the typeset of a classification³⁴, echoing the partiality of situations in situation theory³⁵. Neither is it assumed that the set of types is closed under any logical operation such as negation. However, if the set of types *is* structurally closed under negation then we may dispense with the negative typing relation since $a :^+ \alpha$ iff $a :^- \neg\alpha$ and $a :^- \neg\alpha$ iff $a :^+ \alpha$.

A classification defines a domain of content. In general, how some token is classified by some type is only relative to a particular classification; for example, it is not required by the theory that any two classifications classify the same tokens consistently with one another. This is not surprising considering how Seligman (1990a; 1990b) uses classifications to describe perspectives. For example, two research assistants may be given the task of classifying a set of archeological artifacts by various typological variables such as color, texture, shape, and size. It is frequently the case in such circumstances that there will be slight variations in

³⁴ Cavedon (1995, 33-34) defines a classification somewhat differently. In place of the function k , here defines two functions $k^+ : tok(A) \rightarrow 2^{typ(A)}$ and $k^- : tok(A) \rightarrow 2^{typ(A)}$ mapping tokens to sets of types. Defined in this way, for any given token a and type α it is possible that both $k :^+ \alpha$ and $k :^- \alpha$, an impossibility for the definition of a classification given her and in Barwise and Seligman (1993). Cavedon calls classifications in which there are inconsistent type assignments *incoherent*. Note that this definition is equivalent to that of later channel theory, except that it assumes a kind of closure under negation.

³⁵ This definition is equivalent to a total assignment function $k : tok(A) \rightarrow \{+, -, u\}$ where u stands for undefined, echoing Kleene's three-valued logic.

how each object is classified, depending on now the types are interpreted and on the various conditions in which they are observed. One assistant might classify a particular object as blue and the other classify it as blue-grey. A channel links such classifications together in a way that is intended to allow information about how some particular is classified in one domain to indicate how some particular is classified in another domain. The connections between the particulars of each channel, and the type constraints between them, are modeled again by another classification.

Definition 3.24 (Barwise and Seligman 1994, 259). Let A and B be two classifications. A *bifunction* $f : A \rightrightarrows B$ from A to B is a pair $\langle f^\wedge, f^\vee \rangle$ of functions $f^\wedge : \text{typ}(A) \rightarrow \text{typ}(B)$ and $f^\vee : \text{tok}(A) \rightarrow \text{tok}(B)$. A bifunction $f : A \rightrightarrows B$ is a *homomorphism* if it satisfies the following properties:

1. If $a :^+ \alpha$ in A then $f^\vee(a) :^+ f^\wedge(\alpha)$ in B .
2. If $a :^- \alpha$ in A then $f^\vee(a) :^- f^\wedge(\alpha)$ in B .

A bifunction $f : A \rightrightarrows B$ is an *infomorphism*³⁶ if it satisfies the following properties:

1. If $a :^+ \alpha$ in A then $f^\vee(a) :^+ f^\wedge(\alpha)$ in B .
2. If $f^\vee(a) :^- f^\wedge(\alpha)$ in B then $a :^- \alpha$ in A .

Remark. We will frequently dispense with the superscripts for the component functions of a bifunction since these can be easily distinguished by their signature.

³⁶ The infomorphism of intermediate channel theory is *not* the infomorphism of later channel theory. The infomorphism of later channel theory is a contravariant pair of functions $f^\wedge : \text{typ}(A) \rightarrow \text{typ}(B)$ and $f^\vee : \text{tok}(B) \rightarrow \text{tok}(A)$ satisfying the property that $b : f^\wedge(\alpha)$ iff $f^\vee(b) : \alpha$. Also later channel theory dispenses with the negative typing relation.

We are now ready to define the channels of intermediate channel theory. Barwise and Seligman do not give an explicit definition of a channel in either of their published papers from this period, so we adapt the definition given by Lawrence Cavedon in his PhD dissertation.

Definition 3.25 Channel (Cavedon 1995, 43). Let A and B be two classifications. A channel $\mathbb{C} = A \rightsquigarrow B$ from A to B is a triple $\langle left_C, C, right_C \rangle$ where C is a classification called the *core* of the channel, and $left_C : C \rightrightarrows A$ and $right_C : C \rightrightarrows B$ are bifunction homomorphisms. We call the types of C *constraints* and the tokens of C *connections*. We call the tokens of A *signals* and the tokens of B *targets*. We call the types of A *indicating types* and the types of B *indicated types*.

Remark. We will drop the channel subscript from the left and right bifunction homomorphisms, unless necessary.

It will frequently be useful to represent a constraint γ as a pair of types $T \rightrightarrows T'$ and a connection as a pair of tokens $s \mapsto t$, where $T = left^\wedge(T \rightrightarrows T')$, $T' = right^\wedge(T \rightrightarrows T')$, $s = left^\vee(c)$, and $t = right^\vee(c)$ for some connection c and constraint γ . However, in general there is nothing preventing there being a distinct constraint η for which $left(\eta) = left(\gamma) = T$ and $right(\eta) = right(\gamma) = T'$. Similarly, the same pair of tokens may be associated with more than one connection, each of which might support different (but necessarily logically compatible) constraints³⁷.

³⁷ The necessity follows from the fact that *left* and *right* are bifunction homomorphisms and the fact that our definition of a classification precludes the possibility of the same token being classified both positively and negatively by the same type. Note however that, as we remarked earlier, in Cavedon's work such a possibility is not precluded by his (slightly different) definition of a classification.

The definition of a channel is intended to guarantee the flow of information. In particular, if $c :^+ \gamma$ in C then $left^\vee(c) :^+ left^\wedge(\gamma)$ in A and $right^\vee(c) :^+ right^\wedge(\gamma)$ in B , and if $c :^- \gamma$ in C then $left^\vee(c) :^- left^\wedge(\gamma)$ in A and $right^\vee(c) :^- right^\wedge(\gamma)$ in B (Cavedon 1995, 45)³⁸. At the same time, the definition of a channel provides an extra degree of freedom in modeling imperfect information flow. For example, not every connection in a channel supports every constraint, so that even if the signaling token is of the ‘right’ indicating type the target token may not be of the type indicated by the constraint.

Cavedon gives explicit definitions for several kinds of error identified in Barwise and Seligman (1993): these include pseudo-signals, exceptions, strong exceptions, and weak exceptions.

The definition of a pseudo-signal is somewhat different, but basically equivalent to the definition we gave before.

Definition 3.26 (Cavedon 1995, 46-47). Given a channel $\mathbb{C} = A \rightsquigarrow B$, a *pseudo-signal* for a constraint $\gamma \in typ(C)$ in the core of the channel is a token $s \in tok(A)$ for which $s : left^\wedge(\gamma)$ in A but for which there is no token $c \in tok(C)$ such that $s = left^\vee(c)$.

Example 3.13 Fireflies. Let $\mathbb{C} = A \rightsquigarrow B$ be a channel and let $T \Rightarrow M$ be a constraint of $typ(C)$ where T is the type of situation in which a light signal is of pattern P, and where M is the type of situation in which there is a female *pyralis* interested in mating. Suppose that $left^\wedge(T \Rightarrow M) = T$ and $right^\wedge(T \Rightarrow M) = M$.

³⁸ Barwise and Seligman (1993, footnote 10) write that there are two competing notions of channel under their consideration. The definition we have given here is the stronger one, satisfying what they call the ‘flow principle’: namely the property that if a connection $s \mapsto t$ positively (or negatively) supports a constraint $T \Rightarrow T'$, then s is of type T and t is of type T' positively (or negatively). The weaker version of a channel was intended to tolerate counterfactual constraints, where a connection $s \mapsto t$ positively (or negatively) supports a constraint $T \Rightarrow T'$ but s is *not* of type T .

Let s be a situation in $tok(A)$ of type T , the type of situation in which a signal of pattern P is intercepted. Let us suppose that, in actuality, a predatory mimic is responsible for the signal, and no constraints regarding mimicking predators are in the channel. Then s is a pseudo-signal for the constraint $T \Rightarrow M$. This is *not* an exception to the channel however. It would only be an exception to the constraint if the constraint had actually been in effect. We describe exceptions next.

One innovative difference between early channel theory and intermediate channel theory is the use of a classification at the core of the channel. Since not every token in the core will be classified by the same constraints, errors can also occur because the active channel token is not classified by some constraint when the signaling token suggests that it ought to be.

Definition 3.27 (Cavedon 1995, 47). Given a channel $\mathbb{C} = A \mapsto B$, an *exception* to a constraint $\gamma \in typ(C)$ is a token $c \in tok(C)$ such that $left^\vee(c) : left^\wedge(\gamma)$ in A and $c \not\vdash \gamma$.

We give an example of an exception below.

Example 3.14. Let us take the channel and constraint of the previous example. Suppose that some token c in the core is not classified by the constraint $T \Rightarrow M$, but $s = left^\vee(c)$ and $s : T$. Then c is an exception to the constraint $T \Rightarrow M$.

Example 3.15. Let us define a channel as in our preceding two examples except that tokens are classified additionally by the constraint that $T \Rightarrow P$, where P is the type of situation in which there a predator responsible for the signal. Let $c : T \Rightarrow M$, $c \not\vdash T \Rightarrow P$, $c' : T \Rightarrow P$, and $c' \not\vdash T \Rightarrow M$. Then, c is an exception to $T \Rightarrow P$ and c' is an exception to $T \Rightarrow M$. Note that in this channel, what had

been a pseudo-signal in our first example is not a pseudo-signal in this channel. This underscores the channel-relativity of imperfect information flow.

Intermediate Channel-Theoretic
Analysis of Conditionals

Lawrence Cavendon gives an interesting channel-theoretic account of conditionals in (1995, 1996). A channel-theoretic semantics for conditional statements claims that a conditional statement asserts “that a certain channel $[\mathbb{C}]$ contains a connection $s \xrightarrow{\mathbb{C}} t$ amongst its tokens and a constraint $T \Rightarrow T'$ amongst its types,” (Cavendon 1996, 126)³⁹.

Definition 3.28 (Cavendon 1996, 126). Let K be a classification whose tokens are a set of channels, and whose types are *conditional facts* of the form $\langle \Rightarrow, \Phi, \Psi \rangle$ where Φ and Ψ are Austinian propositions. For a token $\mathbb{C} \in tok(K)$, $(\mathbb{C} : \langle \Rightarrow, (s : T), (t : T') \rangle)$ in K iff $s \xrightarrow{\mathbb{C}} t \in tok(\mathbb{C})$ and $T \Rightarrow T' \in typ(\mathbb{C})$.

Remark. Note that it needs not be the case that $s \xrightarrow{\mathbb{C}} t :_{\mathbb{C}} T \Rightarrow T'$ in order for $(\mathbb{C} : \langle \Rightarrow, (s : T), (t : T') \rangle)$ to hold. If $s \xrightarrow{\mathbb{C}} t :_{\mathbb{C}} T \Rightarrow T'$ were to hold, then by definition of the two channel bifunction homomorphisms, $s : T$ and $t : T'$. But Cavendon wants conditional facts to hold even if they are counter-factual. If s were of type T , then t would be of type T' .

A consequence of this definition is that certain counter-intuitive problems with the material conditional are avoided. For example, conditional facts expressed by conditional statements like:

If Joe likes pepperoni on his pizza then pi is an irrational number.

³⁹ Cavendon makes the simplifying assumption that tokens in the core of the channel can be identified with their endpoints.

where the antecedent is irrelevant to the consequent, need not hold merely because π is necessarily an irrational number (Cavedon 1996, 126-7). Unfortunately, this achievement comes at a price: certain cases of problematic transitivity are not avoided. Suppose that we have the following two conditional statements:

If the switch is ‘On’ and the battery is dead then the switch is ‘on’.

If the switch is ‘on’ then the bulb is lit.

If we were to compose these two propositions then we would get the ridiculous conditional that:

If the switch is ‘on’ and the battery is dead then the bulb is lit.

The problem is that, arguably, one can always define a logical channel such that the first conditional, which is logically true, is supported by any token. If we were to compose this logical channel with a channel whose tokens support the second conditional, then the composition of these two channels is the channel for which there are constraints like that of the third conditional (which may not be supported by any token). However in the classification K of channels, this channel will be classified by related conditional facts. Cavedon proposes to fix this problem by constructing a hierarchy of channel refinements with ever finer implicit background assumptions. A similar notion of channel refinements, and background conditions, finds its way into the mature theory of channels of Barwise and Seligman (1997).

Later Channel Theory

Early and intermediate channel theory was soon scuttled⁴⁰ in favor of a theory having much in common with Jeremy Seligman's theory of perspectives (Seligman 1990a, 1990b, 1991). The standard account of later channel theory is in Jon Barwise and Jeremy Seligman's book (1997). In addition to their book, several recent publications give useful abbreviated introductions to channel theory, and place channel theory within recent discussions on the nature of information. These include van Benthem and Martinez (2008), Mares, Seligman, and Restall (2011), and Burgin (2009).

Barwise and Seligman (1997) motivate their account by stating four principles of information flow: (1) information flow arises out of regularities in distributed systems (Barwise and Seligman 1997, 8), (2) the flow of information involves both types and particulars (Barwise and Seligman 1997, 27), (3) information flow between components of a distributed system arises from the regularities holding of the connections between these components (Barwise and Seligman 1997, 35), and (4) "the regularities of a given distributed system are relative to its analysis in terms of information channels," (Barwise and Seligman 1997, 43).

Barwise and Seligman actually offer *two* related accounts of information flow in their book. The first analysis takes information flow to occur in channels minimally covering distributed systems. The second accounts for information flow through a notion of the logic of a distributed system. Both accounts share similar mathematical foundations. It is to a discussion of some of these that we now turn.

⁴⁰ Mares, Seligman, and Restall (2011, 333) write, "This work on [early] channel theory through to the mid-1990s was, it must be said, a transitional phase. A greater level of generality was reached with the publication of Barwise and Seligman's *Information flow: the logic of distributed systems*."

Classifications and Infomorphisms

The two foundational notions of Barwise and Seligman's channel theory are *classifications* and *infomorphisms*. As in Seligman's theory of perspectives and in intermediate channel theory, a classification is a kind of data structure that classifies tokens or situations by some set of types. The classification relation of later channel theory is simpler than in earlier versions of the theory. For example, there is only one (positive) classification relation. Polarity relations between types must be structurally imposed by the classification relation. We give their definition below:

Definition 3.29 Classification (Barwise and Seligman 1997, 69). A *classification* A is a triple $\langle tok(A), typ(A), \vDash_A \rangle$ consisting of a set, $tok(A)$, of *tokens*, a set $typ(A)$, of *types* classifying the tokens of A , and a binary relation $\vDash_A \subseteq tok(A) \times typ(A)$. If $a \vDash_A \alpha$ then we say that a is of type α in A ⁴¹.

Example 3.16. Let a classification $\mathcal{L} = \langle tok(\mathcal{L}), typ(\mathcal{L}), \vDash_{\mathcal{L}} \rangle$ be such that $tok(\mathcal{L})$ consists of statements in an imperative programming language, $typ(\mathcal{L})$ consists of types of statements such as *declaration statement*, *expression statement*, *selection statement*, and *iteration statement*, and so on, and the classification relation $\vDash_{\mathcal{L}}$ classify statements according to their type.

Infomorphisms link classifications in a kind of part-whole relationship. An infomorphism is a contravariant pair of functions between classifications, one on types and the other on tokens. The tokens and types of a classification may be thought of as situations and situation types (or infons) respectively (Barwise and Seligman 1997, xiv). Infomorphisms, which are distinct from the bifunctions of

⁴¹ We adopt the standard notation used in (Barwise and Seligman 1997).

the same name defined in Barwise and Seligman (1994, 259), replace bifunction homomorphisms as the principal connection between classifications.

Definition 3.30 Infomorphism (Barwise and Seligman 1997, 72). Let $A = \langle tok(A), typ(A), \models_A \rangle$ and $B = \langle tok(B), typ(B), \models_B \rangle$ be any two classifications. An *infomorphism* $f : A \rightleftarrows B$ is a contravariant pair of functions $f = \langle f^\wedge, f^\vee \rangle$, $f^\wedge : typ(A) \rightarrow typ(B)$ and $f^\vee : tok(B) \rightarrow tok(A)$, satisfying the fundamental property that for all types in A and all tokens in B , $f^\vee(b) \models_A \alpha$ iff $b \models_B f^\wedge(\alpha)$.

When no confusion should arise, we will frequently omit the left and right arrows over the function name.

Infomorphisms may be composed (Barwise and Seligman 1997, 75). Given two infomorphisms $f : A \rightleftarrows B$ and $g : B \rightleftarrows C$, then the composition $gf : A \rightleftarrows C$ is the infomorphism given by $(gf)^\wedge = g^\wedge f^\wedge$ and $(gf)^\vee = f^\vee g^\vee$.

We may join classifications together arbitrarily by summing them.

Definition 3.31 Sum of Classifications (Barwise and Seligman 1997, 81). Let A and B be two classifications. The sum of A and B is a classification $A + B$ such that:

1. $tok(A + B) = tok(A) \times tok(B)$,
2. $typ(A + B)$ is the disjoint union of $typ(A)$ and $typ(B)$ given by $\langle 0, \alpha \rangle$ for each type $\alpha \in typ(A)$ and $\langle 1, \beta \rangle$ for each type $\beta \in typ(B)$, such that
3. for each token $\langle a, b \rangle \in tok(A + B)$ $\langle a, b \rangle \models_{A+B} \langle 0, \alpha \rangle$ iff $a \models_A \alpha$ and $\langle a, b \rangle \models_{A+B} \langle 1, \beta \rangle$ iff $b \models_B \beta$.

Remark. For any two classifications A and B there exist infomorphisms $\varepsilon_A : A \rightleftarrows A + B$ and $\varepsilon_B : B \rightleftarrows A + B$ defined such that $\varepsilon_A^\wedge(\alpha) = \langle 0, \alpha \rangle$ and $\varepsilon_B^\wedge(\beta) = \langle 1, \beta \rangle$ for all types $\alpha \in typ(A)$ and $\beta \in typ(B)$ and $\varepsilon_B^\vee(\langle a, b \rangle) = b$ and

$\varepsilon_A^\vee(\langle a, b \rangle) = a$ for each token $\langle a, b \rangle \in \text{tok}(A + B)$ (Barwise and Seligman 1997, 82).

There is an obvious way to generalize this to arbitrary sums of classifications.

Definition 3.32 (Barwise and Seligman 1997, 83). Given an indexed family of classifications $\{A_i\}_{i \in I}$ the sum $\sum_{i \in I} A_i$ of $\{A_i\}_{i \in I}$ is given by: (1) $\text{tok}(\sum_{i \in I} A_i)$ is the Cartesian product of all the sets of tokens in each indexed classification, (2) $\text{typ}(\sum_{i \in I} A_i)$ is the disjoint union of the type sets of each indexed classification, and (3) for each tuple $\vec{a} \in \text{tok}(\sum_{i \in I} A_i)$ and each type $\alpha \in \text{typ}(A_i)$,

$$\vec{a} \models \langle i, \alpha \rangle \text{ in } \sum_{i \in I} A_i \text{ iff } a_i \models_{A_i} \alpha,$$

where a_i is the i th component of \vec{a} . In the obvious category of classifications, these constructions correspond to coproducts.

Channels

A *channel* is a collection of indexed infomorphisms having the same co-domain, called the *core of the channel*. The tokens of the channel core serve as *connections* between the components of the channel.

Definition 3.33 Channel (Barwise and Seligman 1997, 76). A *channel* \mathbb{C} is an indexed family of infomorphisms $\{f_i : A_i \rightleftarrows C\}_{i \in I}$ each having co-domain in a classification C called the *core* of the channel.

Remark. Note that the definition of a channel differs from that of Intermediate Channel Theory.

Given a channel between two classifications A and B there always exists a unique infomorphism from the sum of those two classifications to the core of the channel:

Proposition 3.3 Universal Mapping Property for Sums (Barwise and Seligman 1997, 82). Let A , B , and C be classifications such that there exist infomorphisms $f : A \rightleftarrows C$ and $g : B \rightleftarrows C$. There exists a unique infomorphism $f + g : A + B \rightleftarrows C$ such that $(f + g)(\varepsilon_A) = f$ and $(f + g)(\varepsilon_B) = g$. Equivalently, there exists a unique infomorphism $f + g : A + B \rightleftarrows C$ such that the diagram in Figure 2 commutes.

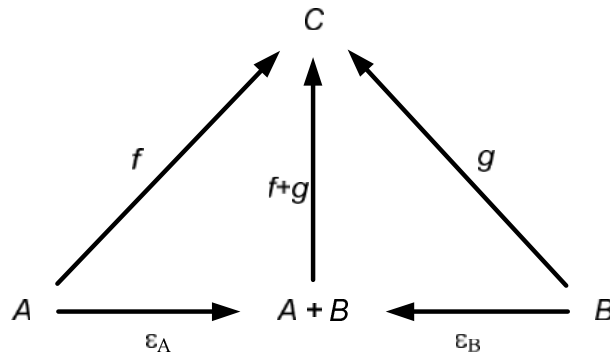


Figure 2. Universal Mapping Property of Sums

Proof. See Barwise and Seligman (1997, 82).

Remark. As remarked upon earlier, the sums of classifications can be generalized to arbitrary indexed families of classifications. Similarly, the universal mapping property of sums can be so generalized. These are the standard universal properties of coproducts.

Distributed Systems

Recall that Barwise and Seligman's first principle of information flow states that information flow arises from regularities in distributed systems. Barwise and Seligman give a simple definition of a distributed system: a distributed system is simply any collection of classifications and infomorphisms between those classifications:

Definition 3.34 Distributed System (Barwise and Seligman 1997, 91). A *distributed system* \mathbb{A} is an indexed family $cla(\mathbb{A}) = \{A_i\}_{i \in I}$ of classifications and a set of infomorphisms $inf(\mathbb{A})$ such that both the domain and the codomain of each infomorphism in $inf(\mathbb{A})$ are in $\{A_i\}_{i \in I}$.

Remark. In category-theoretic terms, a distributed system is just a diagram in the category of classifications.

Given an arbitrary distributed system, so defined, we may obtain an information channel covering that distributed system. In fact, for each distributed system there is a minimal channel covering it, unique up to isomorphism. Thus, for any arbitrary distributed system, we can give an analysis of information flow in terms of the regularities, as set by the classification relations of each classification and the set of infomorphisms.

Definition 3.35 (Barwise and Seligman 1997, 92). A distributed system \mathbb{A} is *covered* by a channel $\mathbb{C} = \{h_i : A_i \rightleftarrows C\}_{i \in I}$ if $cla(\mathbb{A}) = \{A_i\}_{i \in I}$ and for every $i, j \in I$ and each infomorphism $f : A_i \rightleftarrows A_j$ in $inf(\mathbb{A})$ the diagram in Figure 3 commutes. The channel \mathbb{C} is a *minimal cover* of \mathbb{A} provided that it covers \mathbb{A} and for every other channel \mathbb{C}' covering \mathbb{A} there is a unique infomorphism from \mathbb{C} to \mathbb{C}' . In category-theoretic terms, this means that \mathbb{C} is a cocone for the diagram \mathbb{A} .

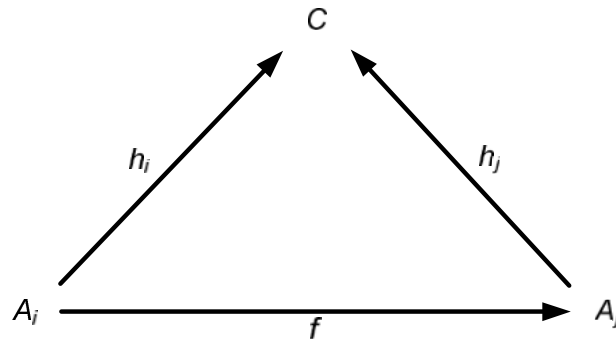


Figure 3. Minimal Cover

Barwise and Seligman (1997, 93-94) show a canonical way in which to obtain the minimal cover of a distributed system from the indexed sum of the classifications in the distributed system. In particular, the core of the channel covering the distributed system is a dual quotient of the indexed sum. The dual quotient of a classification A is a classification whose tokens are a subset of the tokens of A and whose types are R -equivalence classes of the types of A . There is always an infomorphism from a classification to its dual quotient, called the canonical quotient infomorphism. Without going into too many details, the core of the channel covering the distributed system is the dual quotient of the indexed sum whose tokens are those tokens connected by the infomorphisms of the channel, and whose types are the R -equivalence classes of the types from the indexed sum, where R is defined in terms of the infomorphisms of the distributed system. The infomorphism from the indexed sum to the core is simply the canonical quotient infomorphism, as indicated in Figure 4, in which

$$\left(\sum_{i \in I} A_i\right)/J$$

is the dual quotient of the indexed sum and J signifies the invariant it is divided by. In category-theoretic terms, this is the colimit of the diagram \mathbb{A} .

We are now ready to formulate Barwise and Seligman's (1997) first account of information flow.

$$\begin{array}{c} (\sum_{i \in I} A_i) / J \\ \uparrow \\ \sum_{i \in I} A_i \end{array}$$

Figure 4. Canonical Minimal Cover of a Distributed System

Information Flow in Channels

We introduce the notion of a sequent. A sequent $\langle \Gamma, \Delta \rangle$ is a pair of sets of types. A sequent $\langle \Gamma, \Delta \rangle$ is a sequent of a classification A if all the types in either Γ or Δ are in $\text{typ}(A)$.

Definition 3.36 (Barwise and Seligman 1997, 29). Given a classification A , a token $a \in \text{tok}(A)$ is said to *satisfy* a sequent $\langle \Gamma, \Delta \rangle$ of A , if $a \models_A \alpha$ for every type $\alpha \in \Gamma$ and $a \models_A \alpha$ for some type $\alpha \in \Delta$. If every $a \in \text{tok}(A)$ satisfies $\langle \Gamma, \Delta \rangle$, then we say that Γ *entails* Δ in A , written $\Gamma \vdash_A \Delta$ and $\langle \Gamma, \Delta \rangle$ is called a *constraint* of A .

There is an important relationship between the satisfactions of sequents in classifications connected by infomorphisms:

Proposition 3.4 (Barwise and Seligman 1997, 155). Let $f : A \rightleftarrows B$ be an infomorphism. Suppose that $\langle \Gamma, \Delta \rangle$ is a sequent of A and let $b \in \text{tok}(B)$. Then $f(b)$ satisfies $\langle \Gamma, \Delta \rangle$ in A iff b satisfies $\langle f[\Gamma], f[\Delta] \rangle$ in B .

Proof. We prove by equivalences:

$$\begin{aligned}
 f(b) \text{ satisfies } \langle \Gamma, \Delta \rangle &\text{ iff } (\forall \alpha \in \Gamma f(b) \models \alpha) \wedge (\exists \beta \in \Delta f(b) \models \beta) \\
 &\text{ iff } (\forall \alpha \in \Gamma b \models f(\alpha)) \wedge (\exists \beta \in \Delta b \models f(\beta)) \\
 &\text{ iff } b \text{ satisfies } \langle f[\Gamma], f[\Delta] \rangle. \quad \square
 \end{aligned}$$

These considerations lead to the following inference rules (Barwise and Seligman 1997, 38-39), in which constraints may be “moved” between classifications along infomorphisms⁴²:

Definition 3.37. Given an infomorphism $f : A \rightleftarrows C$

$$f\text{-Intro: } \frac{f^{-1}[\Gamma] \vdash_A f^{-1}[\Delta]}{\Gamma \vdash_C \Delta}$$

$$f\text{-Elim: } \frac{f[\Gamma] \vdash_C f[\Delta]}{\Gamma \vdash_A \Delta}$$

Remark. We have dropped the super-script \wedge from f^\wedge in our definition above.

The rule f -Intro preserves validity but does not preserve invalidity, while the rule f -Elim fails to preserve validity, but does preserve invalidity (Barwise and Seligman 1997, 39-40). We explain.

Suppose that $\langle \Gamma, \Delta \rangle$ is a constraint of C . The sequent $\langle f^{-1}[\Gamma], f^{-1}[\Delta] \rangle$ can fail to be a constraint of A in two ways. First, there may be types in Γ or in Δ

⁴² Alternate forms of these two inference rules are presented in (Allwein 2004, 26). Several additional inference rules have been introduced in the literature. For more information see (Martinez 2004) and (Benthem and Martinez 2008).

which are not in the range of f^\wedge . As a simple and illustrative example, consider classifications A and C , such that $\text{tok}(A) = \text{tok}(C)$, $\text{typ}(A) \subseteq \text{typ}(C)$, and the classification relation of A is the classification relation of C restricted to the types of A . There is an obvious infomorphism $f : A \rightleftarrows C$ such that f^\wedge and f^\vee are the inclusion functions on types and tokens respectively. Suppose that $\langle\{\alpha_1, \alpha_2\}, \{\alpha_3\}\rangle$ is a constraint of C , but that $\langle\{\alpha_2\}, \{\alpha_3\}\rangle$ is not a constraint of C . If the type α_1 is not a type of A and so not in the range of the inclusion function f^\wedge , then, assuming that all the other types are in the range of f^\wedge ,

$$\langle f^{-1}[\{\alpha_1, \alpha_2\}], f^{-1}[\{\alpha_3\}] \rangle$$

cannot be a constraint of A , since

$$\langle f^{-1}[\{\alpha_1, \alpha_2\}], f^{-1}[\{\alpha_3\}] \rangle = \langle\{\alpha_2\}, \{\alpha_3\}\rangle,$$

and $\langle\{\alpha_2\}, \{\alpha_3\}\rangle$ has a token $f^\vee(c) = c$ in A not satisfying $\langle\{\alpha_2\}, \{\alpha_3\}\rangle$, for some c in C not satisfying $\langle\{\alpha_2\}, \{\alpha_3\}\rangle$. A look at the rule f -Elim shows that it is designed to avoid this problem by requiring that the constraint be in the range of f^\wedge .

The second way that the pre-image of a constraint $\langle\Gamma, \Delta\rangle$ of C may fail to be a constraint is that the constraint has some counter-example in the token-set of A not in the range of f^\vee . As may be observed, the rule f -Elim cannot avoid this problem, because token level connections are not explicitly taken into account in the inference rule—in contrast to some of the Gentzen calculi of Barwise, Gabby and Hartonas (1994). However, every valid constraint of C is valid for those tokens of A assigned to the tokens of C by f .

Furthermore, suppose that a sequent $\langle\Gamma, \Delta\rangle$ has a counter-example in A . An infomorphism $f : A \rightleftarrows C$ may not be surjective on tokens, and so there may be no corresponding counter-example in the core classification C . Hence the

invalidity of a constraint in A is not preserved by the inference rule f -Intro. However, valid constraints of A will be preserved in C , even if not every valid constraint of C will be given by f -Intro.

We may extend this analysis to a binary channel with infomorphisms $f : A \rightleftarrows C$ and $g : B \rightleftarrows C$. For example, we would like to be able to reason about the tokens of B using what we know about a token of A . The rule f -Intro preserves the constraints of A in the core C , and we would like to pull the constraints of the core back to B using g -Elim. However, any constraints in the core relating the types of A and B via infomorphisms f and g are lost when applying the rule g -Elim. However, this difficult is nicely resolved by the universal mapping property of sums: we simply pull back from the core of the channel to the sum along the infomorphism $f + g : A + B \rightleftarrows C$ using $f+g$ -Elim.

The interesting thing is that the constraints of the core are not valid in the classification $A + B$, but do hold precisely for the pairs of tokens from A and B connected by tokens in C . Thus if a token $a \in tok(A)$ and a token $b \in tok(B)$ are connected by a token $c \in tok(C)$, then a constraint of the core pulled back using $f + g$ -Elim holds for the token (a, b) in the classification $A + B$. In this way, one way in which one's reasoning about a distal token or situation given a proximal token may fail is if one is mistaken in believing that proximal token and the distal token are connected in a channel where the constraint holds.

We may see how all of this relates to Barwise and Seligman's principles of information flow as arising in distributed systems, simply by noting that for any distributed system one can obtain a unique minimal cover, and that there is an infomorphism from the sum of the classifications in the distributed system to the core of the channel. One may think of the sum of classifications as a minimal cover when there are no infomorphisms, i.e. when information in one

classification carries no information about another. The core of the channel represents perfect information about the distributed system. If a sequent $\langle \Gamma, \Delta \rangle$ is a constraint of the core, then that constraint is informational. However, when reasoning about the periphery, e.g., pairs of tokens in the sum, the sequent will not be satisfied by every token. It is guaranteed to be satisfied only by those tokens that are connected to the channel. We end with an example.

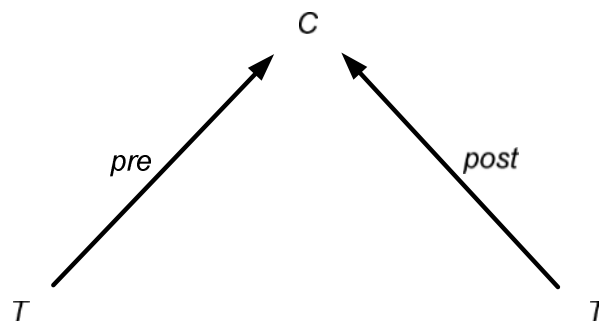


Figure 5. Traffic Light Channel

Example 3.17. Let us suppose that we have a classification of a traffic light system: $T = \langle tok(T), typ(T), \models_T \rangle$. The tokens of our classification will consist of a traffic light at various times and the types of the classification will be *Red*, *Yellow*, and *Green*.

The color of a traffic light carries information about what its next color will be; generally speaking, red transitions to green, green transitions to yellow, and yellow to transitions to red. There may be exceptional cases however. For example, the traffic light might malfunction, or be reset by a technician. To model this we will construct a channel with the diagram in Figure 5 and classification relations as given in Tables 4 and 5. Its infomorphisms are as follows. The infomorphism *pre* maps each color in T to the matching color in $typ(C)$ having

superscript *pre*. The infomorphism *post* maps each color in *T* to the matching color having superscript *post*. On tokens, *pre* and *post* are as described in Table 6.

Table 4. Traffic Light Classification *T*

<i>T</i>	Types		
Tokens	<i>Red</i>	<i>Green</i>	<i>Yellow</i>
t_1	1	0	0
t_2	0	1	0
t_3	0	0	1
t_4	0	0	1

Let us produce the sum $T + T$. For convenience, we re-label each type in the sum with the corresponding labels of the core of the channel. In this way, the principle difference between the core and $T + T$ is that they have different tokens. If we take the sum $T + T$, then by the universal mapping property we have an infomorphism *f* from $T + T$ to *C*. Note that in $T + T$, every possible pairing of tokens from *T* is included, but that for any token in $T + T$:

$$\langle t_i, t_j \rangle = f^\vee(c) \text{ iff } t_i = pre^\vee(c) \wedge t_j = post^\vee(c).$$

Observing that $Red^{Pre} \vdash_C Green^{Post} = f[Red^{Pre}] \vdash_C f[Green^{Post}]$ we can use the inference rule *f*-Elim:

$$\frac{Red^{Pre} \vdash_C Green^{Post}}{Red^{Pre} \vdash_{T+T} Green^{Post}} = \frac{f[Red^{Pre}] \vdash_C f[Green^{Post}]}{Red^{Pre} \vdash_{T+T} Green^{Post}}$$

Thus, by f -Elim, we have that if a traffic light is initially red, then it will be green. But note, that $Red^{Pre} \vdash_{T+T} Green^{Post}$ is not a valid constraint of $T + T$. There will be counter-examples. However, no counter-example token in $T + T$ is assigned to a channel token by f .

Table 5. Traffic Light Channel Core C

C	Types						
	Tokens	Red^{pre}	Red^{post}	$Green^{pre}$	$Green^{post}$	$Yellow^{pre}$	$Yellow^{post}$
c_1		1	0	0	1	0	0
c_2		0	0	1	0	0	1
c_3		0	0	0	0	1	1

Note however that not everything is well. For example, we know that

$$Yellow^{Pre} \vdash Red^{Post}$$

should be a constraint of the system, but we have a counter-example: c_3 . We might suppose that this counter-example is due to a malfunction of some sort. If only we had more tokens we would see that, perhaps. But as it stands, any single counter-example is sufficient to invalidate a constraint.

Indeed, we have an unwanted constraint of the system, one which does not represent a genuine regularity:

$$Yellow^{Pre} \vdash Yellow^{Post}$$

but arises because it happens to have no counter-examples in our classification.

For problems such as these, Barwise and Seligman felt it imperative to develop a

more flexible means of accounting for error. We describe this approach next. We will revisit this example afterwards.

Table 6. Traffic Light Infomorphisms

Tokens	$= pre^\vee(c)$	$= post^\vee(c)$
c_1	t_1	t_2
c_2	t_2	t_3
c_3	t_3	t_4

Theories

Barwise and Seligman generalize these notions in the following way. A *theory* $T = \langle \Sigma, \vdash \rangle$ is a pair consisting of a set of types Σ and a consequence relation \vdash on Σ . A classification is naturally associated with a theory, the theory $Th(A) = \langle typ(A), \vdash_A \rangle$ generated by A whose constraints are exactly those sequents satisfied by all tokens $a \in tok(A)$.

However, not every theory on the types of a classification need be generated by that classification. A theory may be weaker or stronger than its evidence, or a theory might not, for example, include the consequence relation of identity. For this reason, Barwise and Seligman identify a set of structural properties or rules that any reasonable theory on a classification must satisfy.

Definition 3.38 Regular theories (Barwise and Seligman 1997, 119). A theory $T = \langle \Sigma, \vdash \rangle$ is said to be *regular* provided that for all types α , and for all sets of types $\Gamma, \Gamma', \Delta, \Delta', \Lambda \subseteq \Sigma$ the following conditions are satisfied⁴³:

1. *Identity.* $\alpha \vdash \alpha$,

⁴³ Note that for readability we have dropped the set braces, as is customary.

2. *Weakening*. If $\Gamma \vdash \Delta$, then $\Gamma, \Gamma' \vdash \Delta, \Delta'$.
3. *Global Cut*. If $\Gamma, P \vdash \Delta, P'$ for each partition $\langle P, P' \rangle$ of Λ , then $\Gamma \vdash \Delta$.

Barwise and Seligman show that every theory generated by a classification is a regular theory.

A *regular theory interpretation* is a function from the type set of one theory to the type set of another theory that respects the constraints of each. For a theory $T = \langle \text{typ}(T), \vdash_T \rangle$, we let $\text{typ}(T)$ for its type set, and

Definition 3.39 (Barwise and Seligman 1997, 128). Given theories $T_1 = \langle \text{typ}(T_1), \vdash_{T_1} \rangle$ and $T_2 = \langle \text{typ}(T_2), \vdash_{T_2} \rangle$, a *regular theory interpretation* $f : T_1 \rightarrow T_2$ is a function from $\text{typ}(T_1)$ to $\text{typ}(T_2)$ such that for each $\Gamma, \Delta \subseteq \text{typ}(T_1)$

$$\text{if } \Gamma \vdash_{T_1} \Delta \text{ then } f[\Gamma] \vdash_{T_2} f[\Delta].$$

Earlier we introduced the inference rules of f -Intro and f -Elim. We may generalize the rules f -Intro and f -Elim from that of moving individual constraints to the moving of entire theories between classifications. Given an infomorphism $f : A \rightleftarrows C$ and a theory T on C , the generalized form of f -Elim is simply to take the inverse image of the consequence relation of the theory on C . If the theory on C is regular, then its inverse image is also regular.

Definition 3.40 (Barwise and Seligman 1997, 135). Let $T' = \langle \text{typ}(T'), \vdash_{T'} \rangle$ be a regular theory and let $f : \Sigma \rightarrow \text{typ}(T')$ be a function from the set of types Σ to the type set of T' . The *inverse image of T' under f* , written $f^{-1}[T']$, is the theory whose type set is Σ and whose consequence relation satisfies:

$$\Gamma \vdash \Delta \text{ iff } f[\Gamma] \vdash_{T'} f[\Delta].$$

Remark. Barwise and Seligman (135) show that the inverse image of a regular theory is the largest regular theory on the set Σ for which f is a theory interpretation.

The inference rule of f -Intro, relative to an infomorphism $f : A \rightleftarrows C$, corresponds to taking the image of a theory along f . Unfortunately, regularity is not generally preserved when taking the image of a theory. For example, some of the types may not be in the range of f (Barwise and Seligman 1997, 135-136); therefore one must find the smallest regular closure containing the image of the theory. Because we will only use the notion of the inverse image in what follows, and because an adequate definition of the image of a theory involves ideas that we do not have space to explain here, we refer the reader to Barwise and Seligman (1997, 136) for a definition of the image of a theory.

Local Logics

We are now ready to introduce Barwise and Seligman's second, more flexible, account of information flow in distributed systems. This flexibility is achieved by defining information flow in terms of local logics on classifications. A local logic consists of a classification, a regular theory, and a set of normal tokens that satisfy every constraint of the theory.

Definition 3.41 Local Logic (Barwise and Seligman 1997, 150). A *local logic* $\mathcal{L} = \langle tok(\mathcal{L}), typ(\mathcal{L}), \models_{\mathcal{L}}, \vdash_{\mathcal{L}}, N_{\mathcal{L}} \rangle$ comprises a classification $cla(\mathcal{L}) = \langle tok(\mathcal{L}), typ(\mathcal{L}), \models_{\mathcal{L}} \rangle$, a regular theory $th(\mathcal{L}) = \langle typ(\mathcal{L}), \vdash_{\mathcal{L}} \rangle$, and a set of tokens $N_{\mathcal{L}} \subseteq tok(\mathcal{L})$ satisfying all the constraints of $th(\mathcal{L})$, called the *normal tokens* the \mathcal{L} . A logic \mathcal{L} is *complete* when every sequent $\langle \Gamma, \Delta \rangle$ of $cla(\mathcal{L})$

satisfied by every normal token is a constraint of $\vdash_{\mathcal{L}}$. It is *sound* provided that every token is normal.

We see how local logics help answer two problems we had with the previous account. In our previous account, any single exception to a genuine constraint invalidates that information flow. Local logics are not so brittle; the theory of a local logic is impervious to exceptions to the constraints of a theory. Therefore, information flow relative to a constraint still occurs, even when there are exceptions to that constraint. By explicitly including sets of normal tokens in the logic, we differentiate between those tokens for which the information flow is reliable and those tokens for which it is not reliable. Just as importantly, we are able to avoid the Humean empiricist trap and distinguish between mere incidental instances of the satisfaction of a constraint and genuine instances. Furthermore, normal tokens will give us a way to exclude all the spurious tokens created by the sum of two classifications to include only those pairs genuinely connected in the channel. Finally, being explicit about both theories and normal tokens also affords us some flexibility in modeling *imperfect information* about the core of a channel, both in terms of what regularities there are, and in what instances do in fact model those regularities.

There are many details of Barwise and Seligman's theory of local logics that we cannot, for want of space, consider here. We will therefore cut to the gist of the matter, and end with an illustrative example.

Inverse Images of Logics

Local logics may be “moved” between classifications by taking their images or inverse images. We will use the inverse image of a logic at the core of a channel to distribute a logic at its periphery⁴⁴. We give its definition below.

Definition 3.42 (Barwise and Seligman 1997, 166). If $f : A \rightleftarrows B$ is an infomorphism and \mathcal{L} is a local logic with classification B , we define the *inverse image of \mathcal{L}* under f , written $f^{-1}[\mathcal{L}]$, to be the local logic on A such that

1. its theory is the inverse image of the theory of \mathcal{L} and
2. its normal tokens is the set given by:

$$\{a \in tok(A) \mid \exists b \in N_{\mathcal{L}} \text{ for which } a = f(b)\}.$$

Remark. An infomorphism $f : cla(\mathcal{L}_1) \rightleftarrows cla(\mathcal{L}_2)$ between the classifications of two local logics respects those logics if $f^{\wedge}[N_{\mathcal{L}_2}] \subseteq N_{\mathcal{L}_1}$ and if $\Gamma \vdash_{\mathcal{L}_1} \Delta$ then $f[\Gamma] \vdash_{\mathcal{L}_2} f[\Delta]$. Such connections are called *logic infomorphisms* (Barwise and Seligman 1997, 155). The inverse image of a logic \mathcal{L}_2 under an infomorphism g is the greatest logic on the classification in the domain of the logic for which g is a logic infomorphism.

Logic Flows in Distributed Systems

There are several ways in which the information flow in distributed systems may be characterized by local logics. For example, Barwise and Seligman (1997, 183) give a way in which one can define the *distributed logic* of an *information system*, a structure consisting of an indexed family of local logics and a set of logic infomorphisms. Similarly, one can obtain a system-wide logic for a distributed

⁴⁴ Readers interested in a definition of the image of a logic may consult (Barwise and Seligman 1997, 165-167). We will not be using that here.

system by the method described in Barwise and Seligman (1997, 189). For considerations of space, we limit ourselves to an explanation of how we can distribute a logic along a channel, in particular the channel that is the minimal cover of some distributed system.

Let us suppose that we have some distributed system \mathbb{A} such that $cla(\mathbb{A}) = \{A_i\}_{i \in I}$ with infomorphisms $inf(\mathbb{A})$. Suppose that a channel $\mathbb{C} = \{h_i : A_i \rightleftarrows C\}_{i \in I}$ is a minimal cover of \mathbb{A} with commuting diagram as in Figure 6 for each infomorphism $f \in inf(\mathbb{A})$.

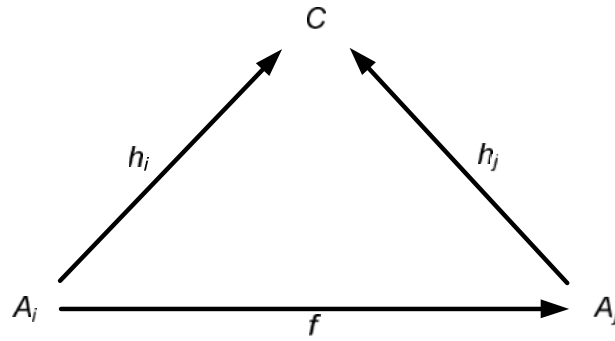


Figure 6. Minimal Cover of Channel

Using the universal mapping property we can obtain the diagram in Figure 7, for each $f \in inf(\mathbb{A})$.

Given some logic \mathcal{L} on the core of the channel, we want to distribute that logic to $\sum_{k \in I} A_k$. As we have already intimated, we can do so by taking the inverse image of the logic on the core of the channel along the infomorphism H , i.e.

$$Log(\sum_{k \in I} A_k) = F^{-1}[\mathcal{L}].$$

Its normal tokens are the set of tokens assigned to the normal tokens of \mathcal{L} by F , and its theory is the inverse image of the theory of \mathcal{L} . The normal tokens are precisely those tokens connected by the infomorphisms of the distributed system at its basis. The distributed logic need not be sound, but is sound for the tokens in the range of F . The logic is complete however because inverse images of logics preserves completeness (Barwise and Seligman 1997, 191).

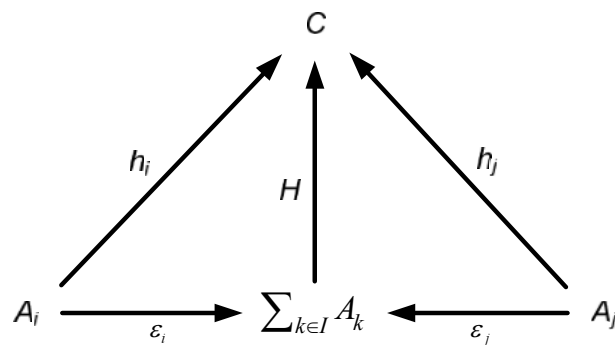


Figure 7. Diagram obtained from Figure 6 using Universal Mapping Property

Our discussion of channel theory has been necessarily brief. There are many important details and additions to the theory that we do not have the opportunity to discuss (such as channel theory's development of a notion of a state space, and its relationship to classifications). We encourage our readers to consult their book. We end our discussion of the details of later channel theory by returning to a previous example.

Example 3.18. We revisit our previous example of the traffic light system. Recall that in our example, we were only partially successful in accounting for imperfect information flow, because there was a counter-example to a genuine regularity. Let us suppose that we have our channel core C and infomorphism

$f : A + A \rightleftarrows C$ as before. We may define a logic on the core of the channel. In this case, our theory in the core will include the constraints:

$$Red^{Pre} \vdash Green^{Post}, Green^{Pre} \vdash Yellow^{Post}, \text{ and } Yellow^{Pre} \vdash Red^{Post}$$

For normal tokens, we have only the tokens c_1 and c_2 . The token c_3 , which had been our counter-example, is not a normal token. If we distribute this logic to the classification $T + T$, we will have as normal tokens in the distributed logic only the tokens $\langle t_1, t_2 \rangle$ and $\langle t_2, t_3 \rangle$. The troublesome token $\langle t_3, t_4 \rangle$, although mapped onto c_3 by the infomorphism f , is *not* a normal token of the distributed logic on $T + T$. Furthermore, since our theory does not include spurious constraints such as

$$Yellow^{Pre} \vdash Yellow^{Post}$$

we have avoided the problem of lacking counter-examples.

Next, we turn to a brief overview of the place of channel theory in the literature.

Applications and Additions to Later Channel Theory

We identify three sorts of connections to the wider literature. Firstly, the notions in channel theory are closely connected to several notions developed in computer science. Secondly, in the years since its initial development there have been a few attempts to supplement the theory. We will discuss these briefly. Lastly, there have been various, sometimes short-lived, attempts to apply channel theory to various sorts of problems. Perhaps most prominent of these is the attempt to apply the theory to problems of ontological alignment in the field of knowledge engineering.

Related Notions

Classifications and infomorphisms are similar to a number of other data structures in the literature. In particular they are similar (Barwise and Seligman 1997, 33) to what are known in the category-theory literature as a *Chu space* and a *Chu morphism* (for an introduction see Pratt 1999)⁴⁵. A Chu space is a generalization of the classification, replacing the \models -relation with a function $k : tok(A) \times typ(A) \rightarrow V$ where V is some set of possible values. The classification relation might be interpreted as the special case in which \models is the characteristic function $\models : tok(A) \times typ(A) \rightarrow \{0,1\}$. A Chu transform is very similar to an infomorphism. Given two Chu spaces $A = \langle tok(A), typ(A), \models_A \rangle$ and $B = \langle tok(B), typ(B), \models_B \rangle$ and Chu transform f from B to A is a pair of functions satisfying the *adjointness* condition, namely that:

$$\models_A(f^\vee(b), \alpha) = \models_B(b, f^\wedge(\alpha))$$

for all tokens in B and all types in A . Chu spaces have been applied to numerous problems in computer science, in particular problems of modeling concurrent automata (Gupta 1994; Pratt 1997).

One interesting piece of work in Chu spaces particularly relevant to issues of information flow is that of van Benthem (2000). van Benthem views Chu spaces as natural models for two-sorted first-order languages with variables over tokens and variables over types. van Benthem asks what information is preserved across Chu transforms. He defines a notion of a *flow formula* in a two-sorted first-order language and proves that all flow formulas are *Chu-preserved*.

⁴⁵ Classifications also are similar to what are known as formal contexts in formal concept analysis. For an introduction to formal concept analysis see (Ganter and Giovanni 1999).

The definition of a flow formula is straightforward. However, we must not confuse the satisfaction relation of first-order logic with the typing relation of channel theory; thus we will now use \models for the satisfaction relation of first-order logic, and use $:$ to indicate the typing relation of channel theory.

A flow formula is a first-order formula generated from the following schema:

$$a : \alpha \mid \neg(a : \alpha) \mid \wedge \mid \vee \mid \exists a \mid \forall \alpha$$

where italicized lower-case letters are tokens and Greek letters are types. Note that existential quantification is only over tokens, and universal quantification is only over types.

Given an infomorphism $f : A \rightleftarrows B$, a first-order formula ϕ is *Chu-preserved* (van Benthem 2000, 2) if

$$B, b, f(\alpha) \models \phi \quad \text{only if} \quad A, f(b), \alpha \models \phi$$

There is a dual notion of Chu preservation in the opposite direction. A dual flow formula has schema

$$a : \alpha \mid \neg(a : \alpha) \mid \wedge \mid \vee \mid \exists \alpha \mid \forall a$$

is dually preserved if (2)

$$A, f(b), \alpha \models \neg\phi \quad \text{only if} \quad B, b, f(\alpha) \models \neg\phi.$$

van Benthem shows that only flow formulas are Chu-preserved. He goes on to give a full first-order preservation theorem. Given an infomorphism $f : A \rightleftarrows B$ a

formula ϕ implies a formula ψ along an infomorphism (Chu transform)⁴⁶ if it is always the case that:

$$B, b, f(\alpha) \models \phi \text{ only if } A, f(b), \alpha \models \psi .$$

van Benthem's preservation theorem asserts that for all first-order formulas ϕ and ψ , ϕ implies ψ along an infomorphism iff there exists a flow formula that is an interpolant between ϕ and ψ .

One final connection to channel theory bears mention, although we regret that we cannot discuss it in any detail. Goguen (2004) shows that many of the core categories of channel theory, including classifications, channels, and local logics, are special cases of a more general category of mathematical object called institutions. Institutions were formulated to address the enormous variety of logics in computer science and other disciplines by formulating a generalization upon the notion of a logic.

Recent Innovations in Channel Theory

For a variety of reasons, the development of channel theory slowed somewhat in the years following the publication of Barwise and Seligman's book. There have been, however, some developments. Martinez (2004) develops an inference engine based on the analysis of the logics of state spaces from Barwise and Seligman (1997), and augments the set of inference rules they proposed. In Seligman (2009), Seligman discusses some of the foundational difficulties in giving a channel-theoretic foundation for information flow by probability. In Moskowitz, Chang, and Allwein (2004) and Allwein (2004), a probabilistic account of channel theory is introduced as a qualitative framework for Shannon

⁴⁶ Note that this is in the opposite direction that is usual in channel theory.

information theory, and applied to problems of information and computer security. Bharath (2008) generalizes upon channel theory by introducing many of the formalisms of fuzzy logic to channel theory to handle graded concepts more naturally. No doubt, much of the work in Chu spaces is applicable to channel theory as well.

Applications

About one third of Barwise and Seligman's book is devoted to delineating potential applications of channel theory to various practical and theoretical problems. These include a channel-theoretic interpretation of Austin's speech acts, problems of the vagueness, common sense reasoning in state spaces, imperfect representations, and even quantum physics. However, there have only been a few scattered attempts by others since then at applying channel theory to problems of practical or theoretical significance⁴⁷.

The most sustained of these have been in the domain of semantic or ontological integration and alignment. The problem of ontological alignment is thus: different knowledge bases may be organized using different ontological categories. However, it is frequently the case that there is some appropriate translation between the categories of one ontology and the categories of another ontology. Finding this translation is called an ontological alignment. The need to align ontologies arises in many practical contexts. For example, when two large companies merge, they may find it in their interests to integrate each company's databases. However, without each database being designed according to the same standard ontology, the integration of such databases requires a great deal of expert effort, resisting automation. Recent developments in the semantic web make the

⁴⁷ That is, if we discount the work applying the theory of Chu spaces.

need for semantic interoperability even more pressing. Kalfoglou and Schorlemmer (2002; 2003) describe a channel-theoretic architecture for ontology alignment. Roughly around the same time the Standard Upper Ontology Working Group of IEEE (<http://suo.ieee.org/IFF/>) was developing a (now-abandoned) standard semantic meta-ontology for semantic interoperability for computing and web platforms. Schorlemmer and Kalfoglou (2005) elaborate upon their former work in developing a theory of the partial semantic integration achieved by agent communication in open environments, while Schorlemmer and Kalfoglou (2008) place their work in the context of Goguen's aforementioned Institutions. Tomai, Prastacos, and Marinos Kavouras (2007) apply channel theory to the integration of geographic ontologies. Schorlemmer (2010) explores the modularity of the information flow framework for ontologies, and Arcas (2010) attempts to situate the semantic alignment of independent agents in their environments.

CONCLUSION

The development of situation semantics and situation theory has had a few—but not many—successes. At the same time it faces many obstacles hindering its widespread adoption. These include an unwieldy formalism, and enormous philosophical difficulties in its modeling. Perry (1998b) notes that situation semantics has had more success in adoption of some of its general themes than in its specific formalisms. Indeed work in situation theory and situation semantics had dwindled by the mid 1990s, largely because, as Devlin (2004, 55) says, “the problems encountered seemed largely intractable given...current knowledge.” Nonetheless, some work continues in situation semantics by Robin Cooper, Jonathan Ginzburg, Angelika Kratzer, and others, as we noted in our introduction.

On the other hand, channel theory has slowly gained more prominence, both in the philosophy and the knowledge-engineering literatures, as we have discussed. Furthermore, its development slowly continues. In part this is because channel theory is similar to other formal developments that have gained prominence, e.g., formal concept analysis, Chu spaces, and Institutions in computer science. However, unless novel and useful applications of the theory are found—and perhaps a prominent champion of the approach emerges—then its future, as well as that of situation theory, remains doubtful. In part this is inevitable, for the approach is too general to be tractable in many applications.

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